

# APPC, Mechanics: Unit $\gamma$ HW 5

Name: \_\_\_\_\_

Hr: \_\_\_\_ Due at beg of hr on: \_\_\_\_\_

U $\gamma$ , HW5, P1

Reference Video: "Atwood's Machine Problem w/Pulley of Non-Negligible Mass and Friction in Its Axle"  
YouTube, lasseviren1, ROTATIONAL MOTION playlist

Here, we will use the figure and solve three different problems. First, we'll review by assuming there is ZERO friction at all. Second, we'll again review, but we'll consider that the pulley has mass and that the rope doesn't slip on the pulley. And finally, we'll tackle the essence of the video above, by adding into the mix the consideration of friction in the axle.

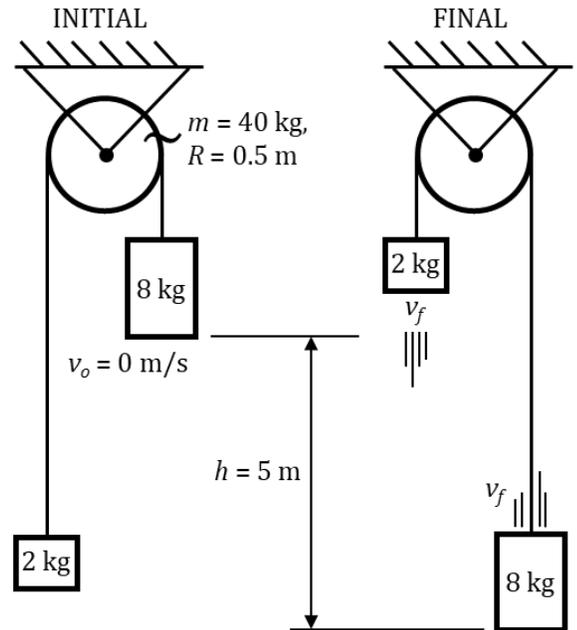
The figure shows the initial and final states. In each case, the system starts from rest and ends after the masses have moved 5 m. Assume the pulley to be a uniform disk. Use  $g = 10 \text{ m/s}^2$ .

A. Assuming no friction at all (i.e., the pulley CANNOT rotate), use conservation of energy to determine  $v_f$ . Feel free to refer back to your work on HW4, P4, Parts B and C. Round your answer to three sig figs.

B. Now, assuming there IS friction between the rope and the pulley (i.e., the pulley WILL rotate without slipping), use conservation of energy to determine  $v_f$ . Refer back to your work on HW4, P4, Parts D-F. Again, it is important to realize that we are assuming ZERO friction between the axle and the pulley.

C. Finally, we consider that there IS friction between the axle and wheel. Ultimately, we wish to find the amount of energy that has been converted into internal energy, due to this friction. To do this, I will TELL you what  $v_f$  is: It is 4 m/s. Firstly, with reference to your Part B answer, why is  $v_f = 4 \text{ m/s}$  a reasonable value for  $v_f$ , in this case?

D. Using conservation of energy, follow the example in the video to determine how much energy has been converted into internal energy between the initial and final states of the system shown, for  $v_f = 4 \text{ m/s}$ .



U<sub>γ</sub>, HW5, P2

Reference Video: "Angular Momentum"

YouTube, lasseviren1, ROTATIONAL MOTION playlist

TRANSLATION

ROTATION

- A. At right, write the appropriate equations for kinetic energy.
- B. Write the bridge equation for displacement. (If needed, refer back to HW2, P2, Part A.)
- C. Show how your Part B answer can be used to justify the claim that a radian is basically...a nothing, i.e., that it is essentially a unitless unit.
- D. Keeping in mind your Part C answer, show that the right sides of your Part A equations yield the same **derived unit**, i.e., a unit that is some multiplication/division combination of one or more SI base units.
- E. Your Part D answer should suggest that  $K_{trans}$  and  $K_{rot}$  are additive. If an object is \_\_\_\_\_ but not \_\_\_\_\_, then it has only  $K_{rot}$  energy; if it is \_\_\_\_\_ but not \_\_\_\_\_, then it has only  $K_{trans}$  energy. But, if it is \_\_\_\_\_ and, at the same time, its \_\_\_\_\_ is \_\_\_\_\_ (the phenomenon of wheels that are in the process of \_\_\_\_\_ is the most common example) then  $K_{total}$  will be the \_\_\_\_\_ of  $K_{trans}$  and  $K_{rot}$ . You've already shown that you've accepted this fact in having completed three previous assignments: \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.

To follow up your Part E answers: In some cases, it is a matter of perspective as to whether something is translating or rotating. Consider the Moon: It doesn't rotate on its axis, in the usual sense; it always keeps the same face directed toward Earth. Suppose we know the Moon's mass is  $M$ , it is a distance  $R$  from Earth's center, and it translates at a speed  $v$ ...

F. In the space at right, draw a picture depicting the last sentence of the above paragraph.

Now, if our perspective is...

| TRANSLATING, but NOT ROTATING   | ROTATING, but NOT TRANSLATING  |
|---|--|
| G. Based on your Parts A and F answers, write an equation for the Moon's $K_{trans}$ . (Easy!)  | H. Write the equation for the moment of inertia of a point mass $M$ going around an axis that is a distance $R$ away. (See HW3, P2, Part A.) |
| <b>Your Parts G and J answers should convince you, for a case such as the Moon, we have to choose EITHER translation OR rotation. We'll get the same answer no matter which one we choose, but we can't choose both. (If we do, we'll get an answer that's 2X too big.)</b> | I. Write the bridge equation for velocity.   |
|   | J. Combine your Parts A, H, and I answers to yield an equation for $K_{rot}$ , in terms of known quantities.                                 |

Momentum and angular momentum are...different. ("Yo' boy's...DIFFent, Miz Gump.")

LINEAR

ANGULAR

- K. At right, write the equation for linear momentum AND its rotational analog.
- L. Show that the right sides of your Part K equations DO NOT yield the same derived unit.
- M. Your Part L answers indicate that, **unlike**  $K_{trans}$  and  $K_{rot}$ ,  $\vec{p}$  and  $\vec{L}$  CANNOT be...

Reference Video: “Angular Momentum”

YouTube, lasseviren1, ROTATIONAL MOTION playlist

A. “The net force on an object (or system) is the time-derivative of the object’s (or system’s) momentum.”

Write the calculus-type equation for the previous statement. Use vector symbols, where necessary.

B. From your Part A answer, we see that: If the net force equals zero, then the object’s (or system’s) momentum RIGHT NOW must compare how...to its momentum, say, later on?

C. The rotational analog of Part A’s opening statement is... “The net \_\_\_\_\_ on an object (or system) is the time-derivative of the object’s (or system’s) \_\_\_\_\_.”

D. Write the calculus equation for your Part C answer. Use vector symbols, where necessary.

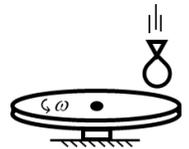
E. Use words to write a statement of the rotational analog for ALL of Part B that follows the colon (:) in that question, i.e., not only the rest of Part B’s question, but also Part B’s answer.

F. At right, write the expanded form of the angular momentum equation. HINT: Your answer should have two ‘equals’ signs, four vector symbols, and one cross product.

G. To review: What does the narrator mean when he uses the term “prime”, i.e., the symbol ‘ ’ ?

A heavy sandbag is about to strike a rotating platform, as shown in the figure.

H. At impact, CIRCLE the direction of the horizontal force on the... platform: ⊗ ⊙  
sandbag: ⊗ ⊙



I. Why will the forces in your Part H answers NOT exert a net torque on the platform/sandbag system?

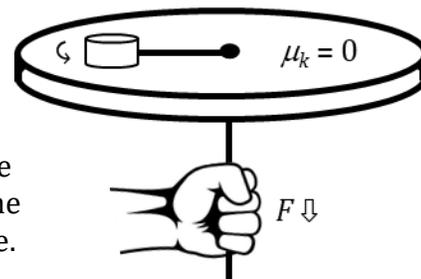
J. For each set of conditions below, circle TWO of the four variables. A large variable indicates a large magnitude for that quantity; a small variable indicates a small magnitude for that quantity.

|                             | INITIAL  | FINAL   |
|-----------------------------|--|---|
| i. Diver, in the air...     | limbs close to torso, spinning quickly<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub> | limbs entended, to enter the water<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub>  |
| ii. Rotating star...        | normal form, shining brightly<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub>          | collapsed into a dense neutron star<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub> |
| iii. Spinning ice skater... | arms stretched out horizontally<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub>        | arms brought in, close to body<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub>      |
| iv. Platform, Parts H-I...  | before sandbag hits platform<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub>           | after sandbag hits platform<br>$I$ <sub>1</sub> $\omega$ <sub>ω</sub>         |

U<sub>γ</sub>, HW5, P4

Reference Video: "Angular Momentum (Part II)"

YouTube, lasseviren1, ROTATIONAL MOTION playlist

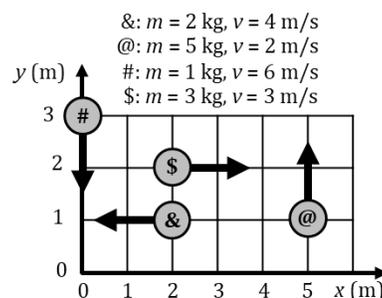


The figure shows a puck attached to a string that passes through a hole at the center of a frictionless table. Initially, a person holds the string lightly, and the puck travels in a large circle. Then, the person pulls down harder on the rope.

- A. What happens to the radius of the puck's motion when the person pulls harder?
- B. When the person pulls harder, does this exert a torque on the puck? Explain your answer.
- C. Based on your Part A answer, how must the rotational inertia of the puck change?
- D. Based on your Part C answer and your previous work (see HW5, P3, Part J), how will the puck's angular velocity change?
- E. What fundamental law or principle of physics led you to your Part D answer?
- F. Go back to your Part A answer. Based on that...Was work done by the person on the puck?
- G. Based on your Part F answer, what must happen to the puck's kinetic energy?
- H. What fundamental law or principle of physics led you to your Part G answer?

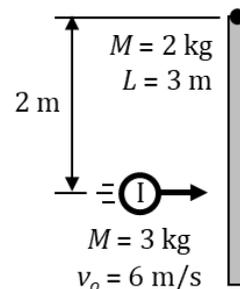
The key point of the next part of the video is that ANY moving mass can be considered to have an angular momentum about ANY random point, via the equation  $\vec{L} = \vec{r} \times \vec{p} = r_{\perp}mv$ .

- I. The masses shown are moving in straight lines on an  $x$ - $y$  grid system. Determine the  $\vec{L}$  of each mass, about the origin, as well as the total  $\vec{L}$  of all the masses put together. Note that there is NO circular motion; nonetheless, we can calculate each mass's angular momentum with respect to any chosen point; here, the origin. Put proper units on your answers, and specify each angular momentum as being CW or CCW.



- i.  $\vec{L}_{\&} =$
- ii.  $\vec{L}_{@} =$
- iii.  $\vec{L}_{\#} =$
- iv.  $\vec{L}_{\$} =$
- v.  $\vec{L}_{total} =$

The figure shows a rod, pinned at one end. A mass of clay approaches the rod. When the two collide, the clay will stick to the rod and the combined mass will begin to rotate.



- J. Determine the kinetic energy of the clay-rod system, prior to impact.
- K. State the type of collision that occurs.
- L. Following the example in the video, determine the angular speed the combined mass will have, immediately after impact.
- M. Use your Parts J and L answers to determine the thermal energy released during the collision.

U $\gamma$ , HW5, P5

Reference Videos: (1) "Rotational Motion Review (Part I)"

(2) "Rotational Motion Review (Part II)"

YouTube, lasseviren1, ROTATIONAL MOTION playlist

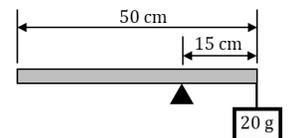
A. An object rotates according to:  $\theta(t) = 3t^3 + 4t^2 + 5t + 6$ . At  $t = 1$  s, determine the object's:

- i. angular velocity
- ii. angular acceleration

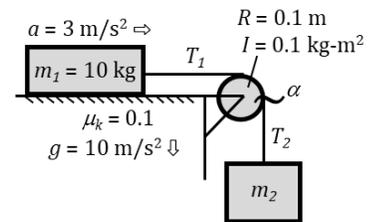
B. If the Part A object is a uniform cylinder rotating about its central axis ( $M = 2$  kg,  $R = 1$  m) find the:

- i. net torque acting on the cylinder at  $t = 1$  s
- ii. rotational kinetic energy of the cylinder at  $t = 1$  s
- iii. angular momentum of the cylinder at  $t = 1$  s

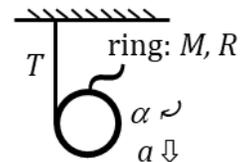
C. The figure shows a system consisting of a uniform rod and a 20-gram mass that is balancing on a pivot. Determine the rod's mass, in grams.



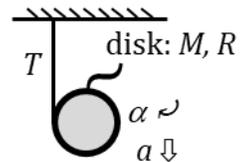
D. Using the figure at right, determine  $T_1$ ,  $\alpha$ ,  $T_2$ , and  $m_2$ .



E. A thin ring is suspended from the ceiling with a string that is wound around the ring. The ring is released from rest. By writing three equations with three unknowns, determine  $T$ ,  $a$ , and  $\alpha$ , in terms of  $M$ ,  $R$ , and the fundamental constant  $g$ .



F. Repeat your work of Part E, but this time you have a uniform disk instead of a ring.



G. Based on your Parts E and F answers, CIRCLE the correct answers below. In which case...

- |   |      |      |            |
|---|------|------|------------|
| i. ...would the object hit the floor first?                               | RING | DISK | IT'S A TIE |
| ii. ...would the string be more likely to snap?                           | RING | DISK | IT'S A TIE |
| iii. ...is there a larger net force on the object?                        | RING | DISK | IT'S A TIE |
| iv. ...is there a larger net torque on the object, about its <i>com</i> ? | RING | DISK | IT'S A TIE |