**APPC, Mechanics: Unit  HW 4** Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 Hr: \_\_\_\_ Due at beg of hr on: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

U, HW4, P1

Reference Video: “Rotational Dynamics”

YouTube, lasseviren1, ROTATIONAL MOTION playlist

 TRANSLATION ROTATION

A. At right, write equations for Newton’s 2nd law. Either form (the

YES-denominator form or the NO-denominator form) is fine.

The figure shows a uniform disk that is nailed to the wall through its *com* such that it can rotate about its *com*. A light string is wrapped around-and-around the disk, and a constant force is applied to the string. Assume that the nail is a frictionless axle.

B. Besides the tension in the string, there are two other forces that act on the disk.

Into the figure, at the proper location(s), draw and label these two other forces.

C. In just a minute, you will apply Newton’s 2nd law for rotation to the disk, BUT...

when you do, you will NOT include the forces from your Part B answer. Why not?

D. Determine the net torque on the disk. Include proper units.

E. Determine the *I* of the disk, about its *com*. HINT: See HW3, P5, Part F.

F. Use Newton’s 2nd law and your Parts D and E answers to determine the angular acceleration of the disk.

G. Is your Part F answer constant for this situation, or not? Explain how you know.

H. If there WERE friction from the axle, what would that do to your Part F answer?



This figure shows a uniform rod that is pinned at one end and free at the other. At the free end, there is an additional suspended mass. The rod-mass system is released from rest, with the rod initially in a horizontal position.

I. Determine *I* for the rod. HINT: Begin with your work on Part I of HW3, P5 and then – because the axis is NOT through the *com* – use the parallel-axis theorem. (Or you could just look up the formula...)

J. Determine *I* for the suspended mass. Refer back to Parts A and C of HW3, P2.

K. The total *I* for a compound system is simply the sum of the *I*s for each part. So,

add your Parts I and J answers to obtain the total *I* of the rod-mass system.

L. Determine the net inital torque on the rod-mass system.

M. Determine the initial angular acceleration of the rod-mass system.

N. Is your Part M answer constant, as the rod descends? Explain your answer.

U, HW4, P2

Reference Video: “Rotational Dynamics (Part II)”

YouTube, lasseviren1, ROTATIONAL MOTION playlist

A. The cylinders at right start from rest. In Case I, there is no friction; in Case II, there IS. On the ‘dashed’ cylinders, use symbols to show what is happening in the ‘dashed’ location. These symbols include a labeled velocity arrow (originating on a cylinder’s *com*) and pointing in the direction of the velocity (or not!) AND

 ⤾ or ⤿ to show rotation (or not!). *Don’t represent anything that isn’t happening*.

B. Notice the straight line ‘painted’ on the top-of-the-ramp cylinders. Onto each ‘dashed’ cylinder, draw an identical line...but at an orientation that is plausible, based on what you said in your Part A answers.

C. To reinforce what you’ve done in Parts A and B... “In Case I, the absence of friction means that the cylinder will merely \_\_\_\_\_\_\_\_\_\_ down the ramp, i.e., it will NOT \_\_\_\_\_\_\_\_ down the ramp. On the other hand, in Case II, the friction between the cylinder and ramp applies a net \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ to the cylinder, which causes the cylinder to \_\_\_\_\_\_\_\_ down the ramp, rather than \_\_\_\_\_\_\_\_\_\_ down the ramp.”

D. One VERY important point when dealing with a rolling object is that, in our FBD, we need to draw all forces where they \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_. A second important point is that we should choose the axis to be through the object’s \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_ \_\_\_\_\_\_\_\_\_. When we do that, two forces that contribute ZERO torque to the rolling object are the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ force and the force of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. This is because the \_\_\_\_\_\_\_\_\_\_ \_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_ of those two forces go directly through the \_\_\_\_\_\_\_\_.

The figure shows a solid, uniform cylinder. When released from rest, it rolls without slipping. As we will see here, analyzing rolling requires us to use BOTH of Newton’s 2nd laws (rotation and translation), as well as a bridge equation that connects the two. Three equations, three unknowns, yada-yada-yada...

E. In the circle below the figure, draw the FBD that applies. HINT: Label the friction force merely as *Ff*. *Make NO reference to  for the rest of this assignment.*

F. Write the Newton’s 2nd law equation for rotation that applies, and simplify it.

HINT: You might want to look back at HW3, P3, Part N.

G. Write the Newton’s 2nd law equation for translation.

There is only one: in the down-the-ramp direction.

H. Write the bridge equation that connects your

Parts F and G answers. (See HW2, P2, Part A.)

I. Substitute your Part H answer into your Part G

answer, and solve for the force of friction.

J. Substitute your Part I

answer into your Part F

answer, and solve for

the angular acceleration. TO BE CONTINUED...

U, HW4, P3

Reference Videos: (1) “Rotational Dynamics (Part II)”

(2) “Rotational Dynamics (Part III)”

YouTube, lasseviren1, ROTATIONAL MOTION playlist

We begin by finishing the our work from the previous assignment, so you’ll need to reference that now.

K. Substitute your Part J answer back into your bridge equation from Part H

to obtain an expression for the translational acceleration of the cylinder.

L. Substitute your Part J answer back into your Part F answer and simplify, to

obtain an expression for the force of friction, in terms of the given quantities.

M. Show that another way to get your Part L answer is to

substitute your Part K answer into your Part G answer.

N. Check that your expressions in Parts J-M yield the correct unit for each Part J \_\_\_\_ Part L \_\_\_\_

quantity you are after. After the units check out, put a 🙂 in each blank. Part K \_\_\_\_ Part M \_\_\_\_

Moving on...

The figure shows a pulley system, with two masses attached. Unlike the Atwood’s machines that we studied previously, this pulley is NOT massless NOR frictionless. This means that the pulley will have a nonzero moment of inertia that you must take into account. (Here, consider the pulley a uniform disk.) ALSO, the rope tensions on either side of the pulley will NOT be equal to each other. Let’s begin...

A. Draw FBDs in the figures below. Label each force you draw. Use *g* to represent the

acceleration due to gravity. (HINTS: The FBD for the pulley has four forces. Also,

because friction is an internal force in the rope-pulley system, do NOT represent friction at this time.)

B. Write the equation for the moment of inertia of the pulley/uniform disk.



C. Write the Newton’s 2nd law equation for translation for the...

 i. ...*M* mass ii. ...3*M* mass

D. Write the Newton’s 2nd law equation for rotation for the pulley. Note that TWO of the four forces in your FBD will (nicely!) NOT appear in the equation.

E. Write the appropriate bridge equation.

F. In Parts C-E, you have four equations and four unknowns. Do any algebraic work necessary to obtain expressions (in terms of given quantities and fundamental constants) for all four unknown quantities.

U, HW4, P4

Reference Video: “Rotational Kinetic Energy”

YouTube, lasseviren1, ROTATIONAL MOTION playlist

 TRANSLATION ROTATION

A. At right, write equations for kinetic energy.

Consider the figure, where the system is released from rest. Your ultimate goal is to derive an expression for the speed with which the 6*M* mass hits the ground, taking into account that the pulley DOES have mass and that there IS friction between the pulley and the rope (but – significantly – NOT any friction in the axle).

B. But first, let’s determine the speed with which the 6*M* mass would hit the ground

if there were NO friction. Document the energies using the chart below.

|  |  |  |
| --- | --- | --- |
| IMPORTANT: We are pre-tending that there’s NO friction in the pulley or axle. | At release of the system, from rest | At the instant the 6*M* mass hits the ground |
| *Ugrav* |  |  |
| *Ktrans* |  |  |
| Total *E* |  |  |

C. Use the bottom line of your chart above to determine the speed *vf*.

D. Okay, but what if there IS friction between the pulley and the rope? (Still NOT the axle, though...) This will cause the pulley to rotate, and THAT means there will be another term in our conservation of mechanical energy table. Complete the table below. (Recall, also, that we consider our pulley as a disk.)

|  |  |  |
| --- | --- | --- |
| IMPORTANT: Now, there’s friction between the pulley and rope, but still none in the axle. | At release of the system, from rest | At the instant the 6*M* mass hits the ground |
| *Ugrav* |  |  |
| *Ktrans* |  |  |
| *Krot* |  |  |
| Total *E* |  |  |

E. Again, the bottom line of the chart will give you the speed *vf* BUT, at the moment, you’re stuck, because there are TWO unknowns and you have only one equation. So, you need the appropriate bridge equation. Write that here. (See HW2, P2, Part A.)

F. Now, combine the bottom line of the chart with your Part E answer to obtain an expression for *vf* .

G. Explain why the magnitudes of your Parts C and F answers differ. Specifically, state the physics behind why one of them is larger than (or smaller than) the other one.

U, HW4, P5

Reference Videos: (1) “Rotational Kinetic Energy”

(2) “Rotational Kinetic Energy (Part II)”

YouTube, lasseviren1, ROTATIONAL MOTION playlist

A. The figure shows a rod pinned to the wall at its left end; its right end is free to move. The rod is released from rest horizontally and the right end descends. In EACH of the three depictions in the figure, draw a dot showing where the rod’s center of mass is. In each depiction, label this dot “*com*”. (Again, ha! “dot-com”...!)

B. Initially, the rod has only \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ energy. As the rod descends, the amount of this initial energy will decrease. Because the rod’s mass is constant – and because gravity is constant – the decrease in this initial energy is due ONLY to the change-in-elevation of the point on the rod abbreviated as \_\_\_\_\_\_. The initial type of energy is converted into \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ energy of the rod, which reaches a maximum when the rod is.....WHERE? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

C. Determine the moment of inertia of the rod shown in the

figure. (It might be helpful to refer back to HW4, P1, Part I.)

D. Use conservation of mechanical energy to determine the rod’s angular speed.

NOTE: Because the axis is NOT moving, you SHALL NOT (“Pass!” Ha, Gandalf...) use a ½ *mvcom*2 term.

i. for  = 30o ii. for  = 90o

In this next problem, the axis of rotation (i.e., the center of the uniform disk) WILL be moving. Therefore, we DO need a *Ktrans* = ½ *mv*com2 term, in addition to the *Krot* = ½ *I*2 term due to the disk rotating, and the total kinetic energy of the rolling disk will be the SUM of these two types of kinetic energy. But, one more thing, before we get into this problem...

E. Suppose we have a wheel rolling along; it has a certain ** and a certain *v*com. Suppose also that we have an identical wheel with the same ** , but it isn’t rolling; it’s just spinning about its central axis and so ITS EDGES have

a particular *v*trans. From the video, how do these *v*com and *v*trans values compare to each other?

**For a rolling thing, the truth of your Part E answer allows you to use a bridge equation**

**to combine its *Ktrans* = ½ *mv*com2 term and its *Krot* = ½ *I*2 term into a SINGLE quantity. 🙂**

F. Use the conservation of mechanical energy (and a bridge equation!) to derive the expression for the rolling speed of the uniform disk when it reaches the bottom of the ramp.