APPC, Mechanics: Unit γ HW 3

Name: ______________________

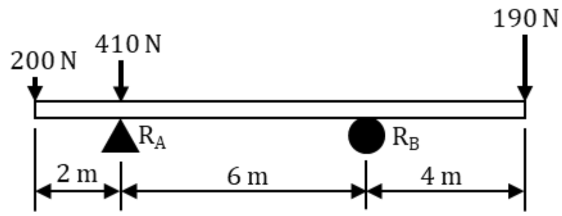
Hr: ___ Due at beg of hr on: ___________

Uγ, HW3, P1

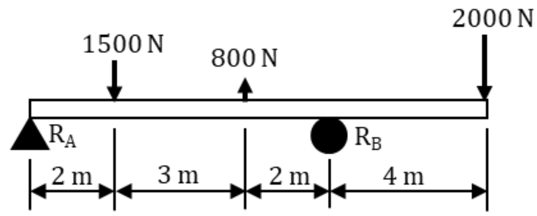
Reference Videos: (1) "Static Equilibrium Problems in Mechanics"
(2) "Static Equilibrium Problems (Part II)"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

A. Use the technique you practiced in HW2, P5, Parts C-H to determine the two support reactions for each of the figures below. As before, consider the weight of the beam as negligible.

i.

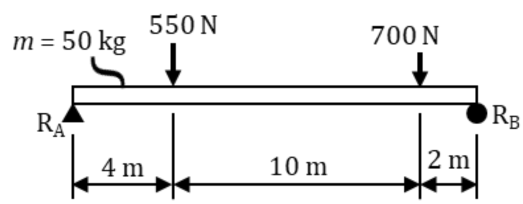


ii.



B. One of the four reactions you obtained in Part A should have stood out from the other three. State how that reaction stood out as being different AND explain the physical meaning of this *different-ness*.

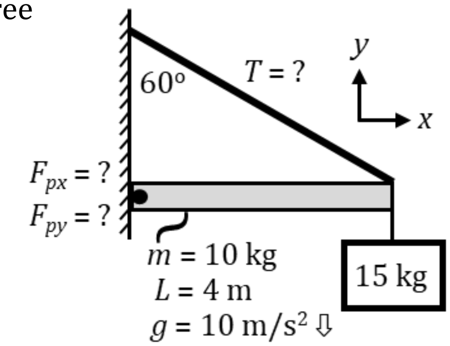
If the weight of the beam is NOT negligible, then you must consider ALL of that weight to act as an additional point load at the beam's center of mass. In solving the following problem, use $g = 10\ \text{m/s}^2$.



C. Into the figure at right, draw in a force vector that represents the beam's weight. Draw the vector in the correct location and point it in the correct direction. Label the vector with its numerical magnitude. Finally, add any necessary dimensions (using two-headed arrows of specified lengths) to show the precise location of your force vector.

D. Now, taking into account your Part C answer, determine the two support reactions for the beam above.

E. Use the technique demonstrated in the second video to determine the three unknown quantities specified in the figure.

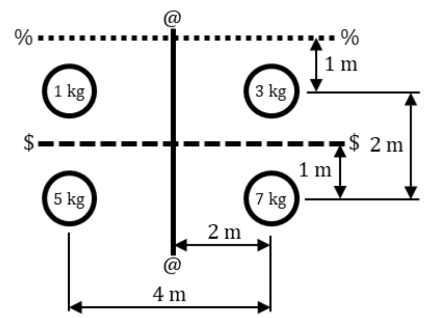


Uγ, HW3, P2
Reference Videos: (1) "Moment of Inertia or Rotational Inertia"
YouTube, lasseviren1, ROTATIONAL MOTION playlist
(2) "Rotational Inertia for a Long Slender Rod" YouTube, lasseviren1

A. Write the general form of the equation for finding the **moment of inertia** of a collection of point masses.

B. The narrator sometimes refers to the moment of inertia (or the **rotational inertia**) as ________________ ____________________________. From your Part A answer, you see that (1) the proper SI unit for moment of inertia is _________ and that (2) the quantity that is MORE influential in determining the value of a certain configuration's moment of inertia is: (CIRCLE) MASS DISTANCE FROM THE AXIS

C. Use your Part A answer to determine the rotational inertia (with units) of the following collections of mass about each of the axes shown.



i. $I_{\text{axis \$-\$}}$ =

ii. $I_{\text{axis \%-\%}}$ =

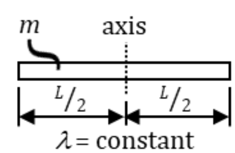
iii. $I_{\text{axis @-@}}$ =

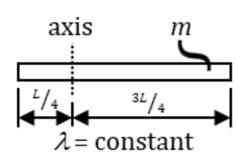
D. Following the pattern from your Part A answer...If we had a VERY tiny mass *dm* that was a distance *r* away from a rotational axis, it would have a VERY tiny moment of inertia *dI*, i.e.,

$dI = dm\, r^2$, which we could rewrite equally well as... $dI = r^2\, dm$

But if we had a BUNCH of tiny *dm* masses, each a unique distance *r* away from the axis, and we wanted the TOTAL moment of inertia *I* from all of those *dm* pieces, then the calculus expression for *I* would be written...how?

In the video, the narrator derives the moment of inertia for a rod of uniform **linear mass density** λ about one axis through the midpoint. (See the figure at right.) As you saw in the video, the answer comes out to be $I = \frac{1}{12} mL^2$. Here, we derive *I* for the same rod, but about the axis shown in the second figure, just below the first.





E. How do you expect your *I* about the new axis to compare to the *I* about the mid-point? Explain briefly. (HINT: Refer back to the last part of your Part B answer.)

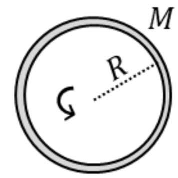
F. λ is the linear mass density, i.e., $\lambda = \frac{mass}{length} = \frac{m}{L}$. Since λ is simply a ratio, it is perfectly valid to write $\lambda = \frac{dm}{dr}$. Solve this last expression for *dm*.

G. Substitute your Part F answer into your Part D answer. Considering the axis to be at $r = 0$, your integration limits can be taken to be from $r = -\frac{1}{4} L$ to $r = +\frac{3}{4} L$. Be sure to include these limits in your answer.

H. Carry out the integration of your Part G answer and simplify to obtain the *I* about the new axis. DON'T forget to substitute back in...the first equation given in Part F.

Uγ, HW3, P3

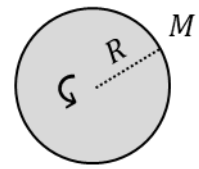
Reference Videos: (1) "Derivation of the Rotational Inertia of a Solid Disk"
(2) "Rotational Inertia for a Cylinder"
YouTube, lasseviren1



A. Write the equation for the moment of inertia of a hoop (or ring) for rotating, wheel-like, about its central axis. See the figure at right.

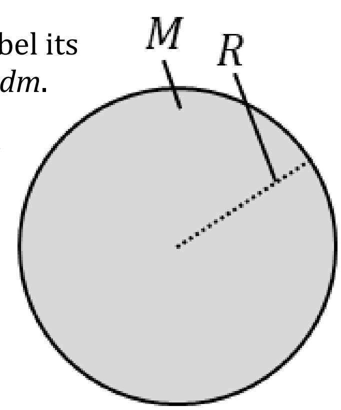
B. An equation that ALWAYS WORKS to find any moment of inertia is your answer to Part D of HW3, P2. Rewrite that equation here.

C. Explain how your Part B answer easily simplifies to your Part A answer. In your response, specifically mention what is true for each of the many *dm* pieces of the hoop.



D. Write the equation for the moment of inertia of a solid disk of uniform mass density for rotating, wheel-like, about its central axis. See the figure at right.

We now will derive your Part D answer. To do so, we must break the disk into many very-thin hoops.



E. Into the large figure of the solid disk at right, draw ONE of the very-thin hoops. Label its distance from the axis as *r*, its very-thin thickness as *dr*, and its very-tiny mass as *dm*.

In the previous assignment, we met the linear mass density λ. Disks, however, don't have *lengths*; they have *areas*. This brings us to **surface mass density** $\sigma = \frac{mass}{area}$.

F. Slight digression: Using the equation just presented, write the expression for the σ of the entire disk, in terms of *M* and *R*.

G. But now, we need an expression for σ for our very-thin hoop, and to do that, we need its mass. What is that mass? (HINT: See Part E.)

H. We also need the hoop's area. If we took the hoop out, cut it as if snipping a rubber band, and unrolled it, it would be a rectangle having length _________ and width ____; thus, its area would be _____________.

I. Substitute your Parts G and H answers into the σ equation given above Part F, to obtain an expression for σ for our very-thin hoop.

J. Solve your Part I equation for *dm*.

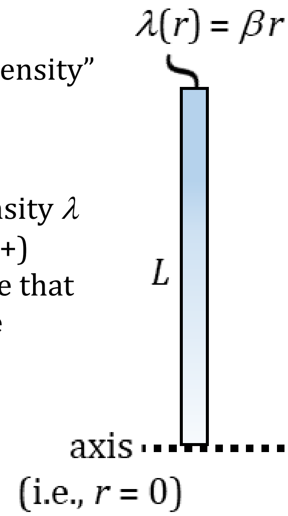
K. Substitute your Part J answer into your Part B answer, and simplify. Check your drawing above, figure out the integration limits for *r*, and be sure to include those limits on your answer.

L. Carry out the integration of your Part K answer.

M. To finish, substitute your Part F answer into your Part L answer. If you've done it correctly, this should yield your response from way back in Part D.

N. In the second video, the narrator derives the moment of inertia for a uniform cylinder. But a cylinder is nothing more than a very thick disk! Therefore, a cylinder's moment of inertia should be given by WHAT equation? Write it here.

Uγ, HW3, P4
Reference Videos: (1) “Rotational Inertia of a Slender Rod with Non-uniform Mass Density”
(2) “Rotational Inertia of a Disk with Non-uniform Mass Density”
YouTube, lasseviren1



First, we derive the moment of inertia of a NON-uniform rod having a linear mass density λ that varies with position *r* according to the known equation $\lambda(r) = \beta r$, where β is a (+) constant. (Perhaps you will notice the shading in the figure, which is meant to indicate that the rod becomes more dense as we approach the top of the page.) We wish to find the moment of inertia for the rod if it were to rotate about the axis shown at the bottom.

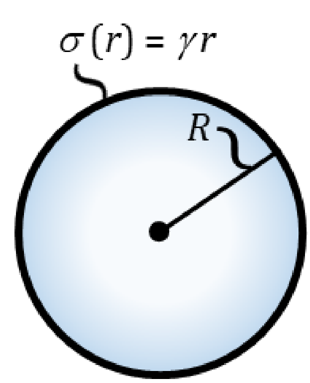
A. Start by writing the equation for finding any moment of inertia.
This was your answer to Part D of HW3, P2 and Part B of HW3, P3.

B. On the rod in the figure, draw in a tiny *dm* element, labeling it as *dm*. Designate that this *dm* element is a distance *r* from the axis AND that the *dm* element has a length *dr*.

C. We need the mass of the *dm* element from Part B. Since $\lambda = \frac{mass}{length}$, we can write $\lambda(r) = \frac{dm}{dr}$. Well, we are given that, for this NON-uniform rod, $\lambda(r) = \beta r$.
Combine these last two equations, and then solve your result for *dm*.

D. Substitute your Part C answer into your Part A answer. Include the limits of integration.

E. Integrate and simplify your Part D answer. You did it!



We now derive the *I* of a NON-uniform disk having a surface mass density $\sigma = \frac{mass}{area}$ that varies with distance from the axis *r* according to the known equation $\sigma(r) = \gamma r$, where γ is a (+) constant. The shading in the figure (assuming you can see it) indicates that the disk becomes more dense as we approach its edges. We wish to find the moment of inertia for the disk as it rotates about its central axis.

F. On the disk, draw in a tiny *dm* ‘hoop’ element, labeling it as *dm*.
Show that the element has a radius *r* AND that it has the thickness *dr*.

G. We need the mass of the *dm* ‘hoop’ element from Part F. Since $\sigma = \frac{mass}{area}$, we can also write $\sigma(r) = \frac{dm}{tiny\text{−}hoop\ area}$. Well, we know the left side of that equation, since $\sigma(r) = \gamma r$.
For the denominator on the right side....If we took the hoop out, cut it as if snipping a rubber band, and unrolled it, it would be a rectangle of length ________ and width ___; thus, its area would be ___________.

H. Combine the two $\sigma(r)$ equations in Part G AND the last equation in Part G...and then solve your result for *dm*.

I. Substitute your Part H answer into your Part A answer (which, of course, still applies, since it ALWAYS does...). Include the limits of integration.

J. Integrate and simplify your Part I answer. Done!

Uγ, HW3, P5
Reference Video: "The Parallel Axis Theorem"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

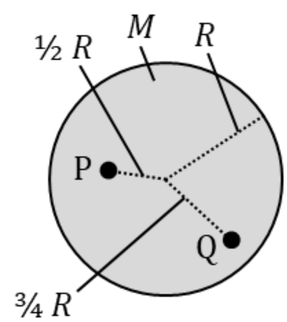
A. The **parallel-axis theorem** allows you to do what?

B. Write the equation for the parallel-axis theorem.

C. In your Part B answer, describe what is meant by the term that is squared. (Depending on the source, this term is given various labels, which is why I just refer to it as "the term that is squared").

D. Look at your Part B answer again and then CIRCLE your answers below.

"The rotational inertia through any non-*com* axis will always be LARGER SMALLER than the rotational inertia through a *com* axis. In other words, the rotational inertia through a *com* axis represents a MINIMUM MAXIMUM rotational inertia for any object about a given *com* axis."



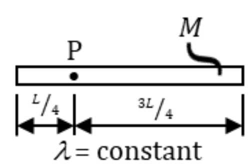
The figure at right shows a uniform disk that is to be rotated about a point that is NOT through its *com*. If you can imagine stabbing a cardboard pizza disk with a pencil at Point P (or, in just a minute, Point Q), and then holding the pencil and whirling the disk around on it...that's essentially what we're doing here.

E. Before we start, do you expect I to be larger about Point P, or about Point Q? Explain.

F. Okay, here we go. First, determine I_{com} for the disk.
HINT: Look back at Part D (and M) on HW3, P3.

G. Use the parallel-axis theorem to find I about Point P. Show work and fully simplify.

H. Now, determine the moment of inertia about Point Q. Show your work, simplify your answer, and then comment on your answers to Parts E, G, and H.



The figure shows a uniform rod being rotated about a point halfway between the center and one end. Rulers often have multiple, pre-drilled holes: If there were a hole at Point P and you put a pencil there and whirled the ruler around...that's what's happening.

I. Determine I_{com} for the rod. HINT: Look back in the commentary prior to Part E on HW3, P2.

J. Use the parallel-axis theorem to determine I for this situation. Show work and fully simplify.

K. Look back at HW3, P2, Part H. How does your answer THERE compare with your Part J answer HERE?