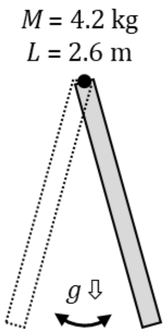
APPC, Mechanics: Unit δ HW 4

Name: ________________________________
Hr: ___ Due at beg of hr on: __________

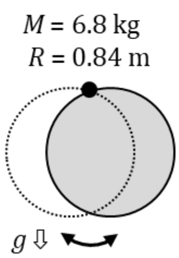
Uδ, HW4, P1
Reference Video: "Physical Pendulum (Part II)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

Determine the period of oscillation for each pendulum below. All objects are uniform masses. Show your work. BOX in your answers, and round them to THREE sig figs. Use 9.8 m/s^2 for g. In determining moments of inertia, it may be helpful to look back at your work from Uγ, HW3, P2 through P5.

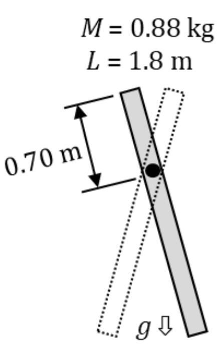
A. Solid rod, pinned at one end



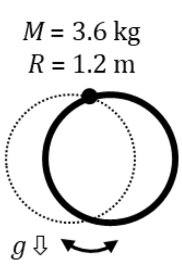
B. Solid disk, pinned at an edge



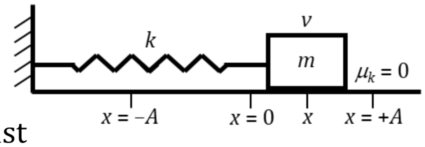
C. Solid rod, pinned at location shown



D. Thin ring, pinned at an edge



Uδ, HW4, P2
Reference Video: "Vertical and Horizontal Harmonic Oscillators"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

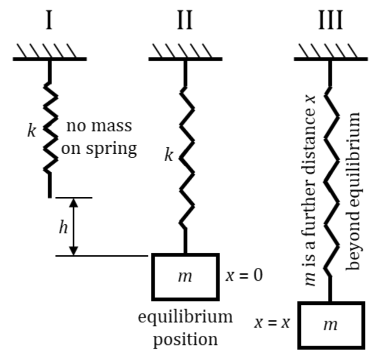


Many students are comfortable dealing with horizontal mass-spring systems, but get confused if the system is oriented vertically. We will attempt to overcome this potential hang-up here.

A. With reference to the figure above-right... Draw an FBD of m when it is at the location x. Employ Hooke's law, putting the associated expression into the FBD.

B. Write a NO-denominators Newton's 2nd law equation for the x-direction.

C. (EASY!) How does the magnitude of the net force on m compare to the magnitude of the elastic force in the spring?



Let's take the same system and turn it vertical, so k and m are the same as before. In State I, no mass hangs from the spring (which we assume is massless). In State II, m hangs from the spring at equilibrium, i.e., State II is how the system would look if m were simply hanging, at rest. You see that, in State II, m's weight has extended the spring a distance h. State III shows m an additional distance x below the equilibrium position. Basically, we have set m to oscillating up and down, and State III is just an instantaneous snapshot showing when m happens to be at the NON-maximum displacement x.

D. Draw an FBD of State II and write a NO-denominators Newton's 2nd law equation. Obviously, any time m is in State II (whether its $v = 0$ or not), m will NOT be accelerating.

E. Draw an FBD of State III and write an unsimplified, NO-denominators Newton's 2nd law equation. HINT: Your answer should have one (+), one (−), and one set of ( ).

F. Get rid of the ( ) in your Part E answer by employing the distributive property.

G. Substitute your Part D answer into your Part F answer and simplify.

H. How do your Parts B and G answers compare?

I. Your Part H answer says that, whether you have a horizontal- OR a vertical mass-spring system, the NET force on m when it is a displacement x from equilibrium always boils down to $F_{net} = ma =$ _____. However, the ELASTIC force in the spring for a horizontal system is equal to _____ (HINT: See your FBD in Part A) while, for a vertical system, the elastic force is equal to _________ (HINT: See your FBD in Part E). Furthermore, since *equilibrium* means something to the effect of 'the location where a system could remain motionless, indefinitely, without changing', then a horizontal system is in equilibrium at $x =$ ____, whereas a vertical system is in equilibrium, in our case above, at $x =$ ____. And because the elastic force is NOT the same for horizontal versus vertical, as well as because the equilibrium position is NOT the same, neither will the ________________ for the two scenarios be the same.

Uδ, HW4, P3

Reference Videos: (1) "Simple Harmonic Motion Review (Part I)"
(2) "Review of Simple Harmonic Motion (Part II)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

A. A position-time equation for SHM is : $x(t) = 3\cos(\pi t)$. Assume standard SI units. Determine each of the following. On Parts iii through viii, include units. On Parts vii and viii, round to three sig figs.

i. velocity-time equation

ii. acceleration-time equation

iii. amplitude of the motion

iv. angular frequency of the motion

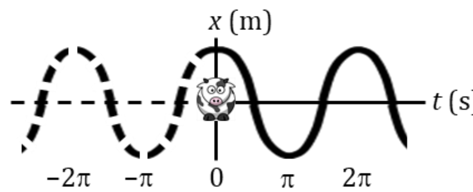
v. frequency of the motion

vi. period of the motion

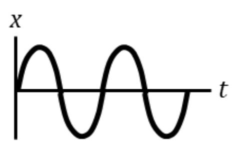
vii. maximum speed

viii. maximum magnitude of acceleration

We now deal with finding the **phase shift** ϕ from the graph of a system oscillating in SHM. You already know that the general form of the displacement equation for such a system is $x(t) = A\cos(\omega t + \varphi)$. If the given graph looks like the right side of the one at right (the non-existent, negative-time part of the curve is shown in dashes), then we know that ϕ = 0. If the graph DOESN'T look like that, then ϕ is NOT zero. So then, what IS ϕ? Here's what you do: ***MOOve the Cosine Cow along the time axis of the figure at right until her new location corresponds to t = 0 for the graph you are given.*** If you moved her to the left, then ϕ is minus-something; if you moved her to the right, then ϕ is plus-something. For example, take the graph at right:

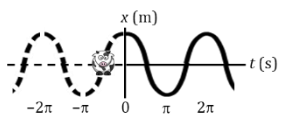


"MOOve the Cosine Cow; She'll graze to find the phase!"



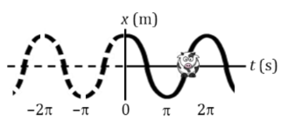
EITHER form of the equation is valid, as they are graphically identical.

MOOving the Cosine Cow to the left yields:



Thus, ϕ is (−) $\pi/2$, and our equation is: $x(t) = A\cos(\omega t - \frac{\pi}{2})$.

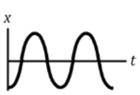
MOOving the Cosine Cow to the right yields:



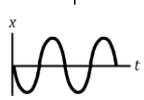
Thus, ϕ is (+) $3\pi/2$, and our equation is: $x(t) = A\cos(\omega t + \frac{3\pi}{2})$.

B. Write both the −ϕ and +ϕ forms of the displacement-time equation, as shown above, for each graph below. NOTE: For simplicity, assume that all ϕs are even multiples of ¼ π.

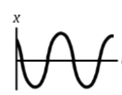
i.



ii.

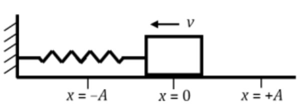


iii.

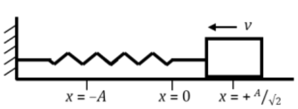


C. *Now, relate equations/graphs/real-life scenarios...* Assuming the mass starts (i.e., t = 0) at each location below, write BOTH forms that ϕ could take. (In one case, ϕ is just ZERO.) Make any ϕ a multiple of ¼ π.

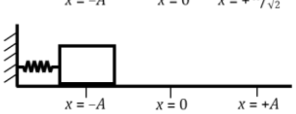
i.



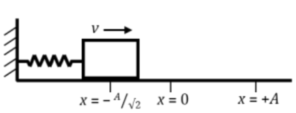
ii.



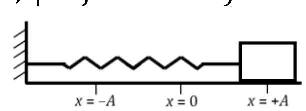
iii.



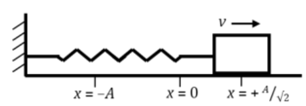
iv.



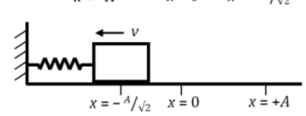
v.



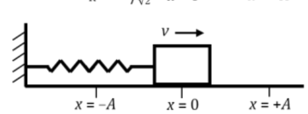
vi.



vii.



viii.



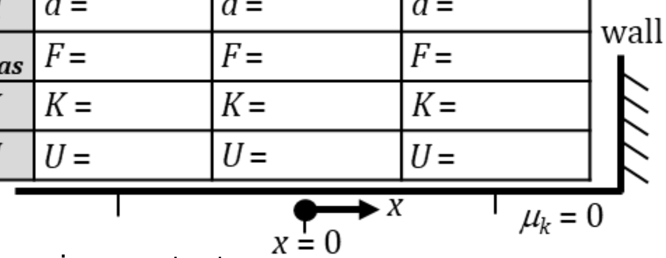
Uδ, HW4, P4

Reference Videos: (1) “Review of Simple Harmonic Motion (Part III)”
(2) “Review of Simple Harmonic Motion (Part IV)”
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

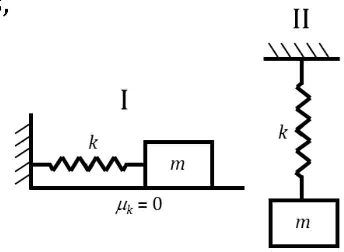
A. The figure shows a wall and a frictionless surface. Although NOT shown in the figure, assume a spring connects the wall and a mass, which oscillates between the tick-marks shown below the surface. Your task is to fill in each of the 18 white boxes to show the value of each quantity at that location. Here are your choices:

zero **(+) max** **(–) max** **(+/–) max**

$\boldsymbol{x}$	$x =$	$x =$	$x =$
$\boldsymbol{v}$	$v =$	$v =$	$v =$
$\boldsymbol{a}$	$a =$	$a =$	$a =$
$\boldsymbol{F_{elas}}$	$F =$	$F =$	$F =$
$\boldsymbol{K}$	$K =$	$K =$	$K =$
$\boldsymbol{U}$	$U =$	$U =$	$U =$



B. The two oscillating systems shown have identical masses, spring constants, and amplitudes. Which system has... (CIRCLE your answers)



i. more energy I II IT’S A TIE

ii. the mass achieving a greater v_{max} I II IT’S A TIE

iii. a greater maximum F_{elas} in the spring I II IT’S A TIE

C. Write the equation for the period of a mass-spring system.

D. Based on your Part C answer, which of the Part B choices has the greater period? I II IT’S A TIE

E. Based on your Part C answer, if we increase the amplitude of either case in Part B, the periods will: INCREASE DECREASE STAY THE SAME

F. Based on your Part C answer, if we set up the Part B cases on the Moon, the periods will: INCREASE DECREASE STAY THE SAME

G. Based on your Part C answer, list two variables that, if changed, will change the period.

H. In Part B’s Case II... Suppose we hook the mass onto the unstretched spring and drop the mass from rest in gravity g, and we want to know the total distance h the mass will fall before the spring stops it.

i. What law or principle of physics should be used to answer this question?

ii. Employ your Part Hi answer to find an expression for the total falling distance h. Show your work.

iii. Show that your Part Hii answer is dimensionally consistent, i.e., that the units on the right side do, in fact, reduce to the proper unit for h.

iv. According to the video, the equilibrium distance (let’s call it e) is equal to _____ h.

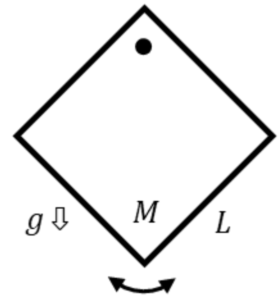
v. Combine your Parts Hii and Hiv answers to obtain an expression for e.

vi. Based on your Part Hv answer, write an expression for the amplitude of the mass’s motion.

vii. Write an expression for the angular frequency of the mass’s motion.

Uδ, HW4, P5

Reference Videos: (1) "Review of Simple Harmonic Motion (Part IV)"
(2) "Review of Simple Harmonic Motion (Part V)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



The figure shows a uniform square plate that will rotate on the axis shown in a gravity field g. The square plate has mass M and side length L.

A. Into the figure, draw and label a dot that represents the plate's center of mass.

B. Assume that the distance between the center of mass and the axis is ½ L. Using dimension lines and a two-headed arrow, represent this information in the figure.

C. The rotational inertia *about the com* of any uniform rectangular plate having length l and width b is given by $I_{com} = \frac{1}{12} M\,(l^2 + w^2)$. In terms of M and L, derive an expression for the I_{com} of the plate in the figure.

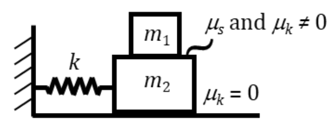
D. But this plate will be rotating about the axis shown, NOT rotating about its *com*.

i. What principle or theorem of physics should be used to address this issue?

ii. Employ the information from Part B as well as your Part Di answer to determine the plate's rotational inertia about the axis shown.

E. This oscillating plate will be what type of pendulum? (CIRCLE) SIMPLE PHYSICAL

F. Based on your Part E answer, employ the proper equation to derive an expression for the period of this pendulum.



G. With reference to the figure, the narrator ultimately derives an expression for the largest displacement x that the system can handle without there being any ______________ between m_1 and m_2. He also makes the important point that the ___________ ______________ force between the masses equals zero at the _____________________ position. This is because, at that position, the two-mass system is NOT ______________________. Therefore, m_1 is also NOT ______________________, which means that the _____ ________ on m_1 at that point is _______ ...and the _____ ________ on m_1 at that point (and all others) is, in fact, the ___________ ______________ force.

As the last video shows, the displacement satisfying the condition of Part G is $x = \frac{\mu_s g}{k}(m_1 + m_2)$. If we pull the spring farther to the right than x, what will m_2's acceleration be when it reaches location x...?

H. Let's first deal with m_1. Because m_1 will be sliding, there will be a kinetic friction force on it. At right, draw an FBD showing the three forces on m_1. Then, use Newton's 2nd law to find m_1's acceleration a_1.

I. Draw an FBD for m_2 when it is at location x. Show all five forces, two of which are INTERNAL from your Part H answer. (Thus, they are equal and opposite to how they appear in Part H.) Then, write a Newton's 2nd law equation; leave it unsimplified.

J. Substitute the expression for x given above Part H into your Part I answer and solve for a_2.

K. Based on your Parts H and J answers, do both masses have the same acceleration when m_2 is at location x? YES NO