APPC, Mechanics: Unit δ HW 3

Name: ______________________________

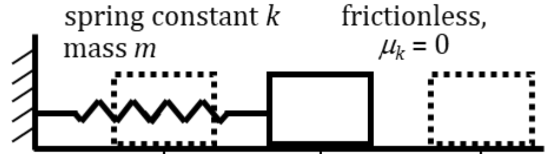
Hr: ___ Due at beg of hr on: __________

Uδ, HW3, P1

Reference Videos: (1) "Kinematics of Simple Harmonic Motion (Part III)"
(2) "Dynamics of Simple Harmonic Motion"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

A. In UCM and SHM, the **period** T is essentially the number of ____________ for one ________, i.e., the ______ for one _________ ______. (So, obviously, the best SI unit for period T is abbreviated ___.) The **frequency** f is the number of __________ per ___________, and is often measured in the unit ________, which is abbreviated ____. A mathematical equation that relates period T to frequency f is ________. Specifically in UCM or SHM, the variable ω is called the _____________ _________________ and has the unit of ____________ per ___________. In the sinusoidal curves that we met in the last assignment, one complete cycle of UCM or SHM is equivalent to ____ radians of angular displacement. Therefore, if we focus on the period of a single cycle, we see that ω = _____; if we focus on frequency, then ω = _______ .

The figure at right shows a mass that oscillates in SHM.



B. Below the figure, draw an FBD of the mass when it is located at some distance x between $x = 0$ and $x = +A$. All forces (one of them is F_{elas}) should originate on the *com* of the object.

C. Hooke's law is $F_{elas} = -k\,x$. The elastic force is one type of **restoring force**, which is any force whose magnitude is ____________ ___________________ to the object's (or system's) __________________.

D. The (−) sign in Hooke's law indicates WHAT?

E. Based on your FBD, write an x-direction, Newton's 2nd law equation. Substitute into it the Hooke's law equation given in Part C, then solve your equation for the mass's acceleration.

F. Into your Part E answer, substitute suitable answers from HW2, P5, Part I. Solve for angular frequency.

G. Use your Part F answer to determine the angular frequency of the mass-spring systems given below.

i. m = 2 kg
k = 32 N/m

ii. m = 24 kg
k = 6 N/m

iii. m = 100 kg
k = 4900 N/m

H. Use an equation you wrote in your Part A answers to determine the period for each of the systems in Part G. Round your answers to two places past the decimal, and include the correct unit.

i.

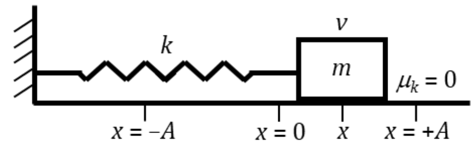
ii.

iii.

I. Combine your Part F answer with the equation you used in completing Part H to derive a (hopefully) recognizable equation for the period of a mass-spring system.

Uδ, HW3, P2
Reference Video: "Simple Harmonic Motion and Energy Conservation"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

The figure at right shows a mass oscillating in SHM. Answer the questions based on the variables shown in the figure.



A. CIRCLE all types of energy the system has at each location, then write an equation expressing the total energy at that point.

i. $x = -A$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=-A} =$

ii. $x = 0$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=0} =$

iii. $x = x$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=x} =$

iv. $x = +A$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=+A} =$

B. Given that $\mu_k = 0$, it is a safe assumption that mechanical energy is ______________________ in this system.

C. In terms of k, A, m, and v, derive an expression for the position x where the mass has the speed v.

D. Show that your Part C answer gives you one or more recognizable locations x when $v = 0$.

E. In terms of A, x, m, and k, derive an expression for v when the mass is at location x.

F. Check your work up to this point by plugging your Part D answers into your Part E answer. What result do you obtain?

G. Using your Part E answer...At what location x will v be maximized?

H. Obtain an expression for $|v_{max}|$ by plugging your Part G answer into your Part E answer.

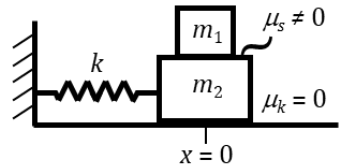
I. Substitute your answer from Part F of HW3, P1 into your Part H answer.

J. Comment on how your Part I answer corresponds to what your HW2, P5, Part I answer has to say about v_{max}.

K. When we maximize the stretch (or compression) of a spring, we also maximize the _______________ force exerted by the spring, in accord with the equation of ______________ law, which is _____ = ______. And for a mass-spring system oscillating in ______________ ___________________ motion, a maximizing of the force above will also go along with a maximizing of the mass's _________________________, according to the equation of ______________ _____ law, which is _____ = ____.

Uδ, HW3, P3

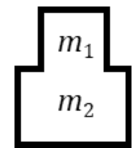
Reference Videos: (1) “Harmonic Oscillator with Two Objects”
(2) “Springs in Series and Parallel”
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



In the figure above-right, we wish to determine the maximum displacement x such that m_1 does NOT slide, relative to m_2. There IS friction between the masses, but none between m_2 and the surface. For maximum displacement, it follows that we must max out the static friction force between m_1 and m_2 .

A. Let us displace the system this maximum amount $+x$, i.e., to the right. (Any larger displacement would give an acceleration too large, and m_1 would slide.) Using Hooke’s law, write an expression for the elastic force F_{elas}.

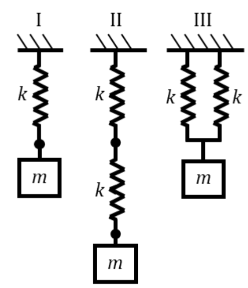
B. Considering ($m_1 + m_2$) as a combined mass, use the figure at right to draw an FBD of this combined mass. Then, using your FBD and your Part A answer, write a Newton’s 2nd law equation and solve it for the acceleration of the two-mass system.



C.

Now, consider only m_1. Draw an FBD using the figure at left. Again, write a Newton’s 2nd law equation and solve it for the acceleration of m_1.

D. If the two masses are to stay together, any part of the system must have the same acceleration, i.e., your Parts B and C answers must be equal to each other. Do that, and solve for x.



E. Based on the figure at right, CIRCLE the correct answers to these conceptual questions about springs. Which system:

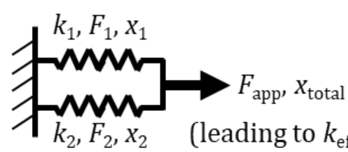
i. ...is the “tightest”? I II III

ii. ...is the “flopsy-est”? I II III

iii. ...will take the most time for one oscillation? I II III

iv. ...will take the least time for one oscillation? I II III

Now, we turn to finding the effective spring constant k_{eff} for multiple-spring systems. Consider the figure, where a force F_{app} is applied to the system shown, resulting in forces and displacements in both springs.



(leading to k_{eff} for the two-spring system)

F. The springs shown are in: (CIRCLE) SERIES PARALLEL

G. When F_{app} is applied, which TWO choices will be true? (CIRCLE <u>TWO</u>)

I. $x_1 + x_2 = x_{total}$ II. $x_1 > x_2$ III. $x_1 < x_2$ IV. $x_1 = x_2 = x_{total}$, i.e., x for all parts of the system

V. $F_1 + F_2 = F_{app}$ VI. $F_1 > F_2$ VII. $F_1 < F_2$ VIII. $F_1 = F_2 = F_{app}$, i.e., F for all parts of the system

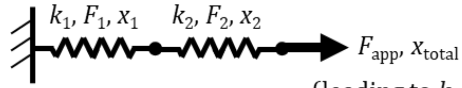
H. One of your Part G answers should have a (+) symbol. Use Hooke’s law to transform that answer’s equation FROM the *one type of variable that it already has* INTO *the other two types of variables that are in Hooke’s law*.

I. Your other Part G answer should have all (=) signs, i.e., NOT three discrete variables, only one. Substitute this ONE variable into your Part H answer, then simplify/cancel as needed. Show your work.

J. If you had three springs configured as above (with spring constants k_1 , k_2 , and k_3), what would the equation be to find k_{eff} for the system?

Uδ, HW3, P4

Reference Videos (1) “Springs in Series and Parallel”
(2) “VID00129 (Period of a Simple Pendulum)”
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



(leading to k_{eff} for the two-spring system)

The figure shows a force F_{app} applied to the system, resulting in forces and displacements in both springs.

A. The springs shown are in: (CIRCLE) SERIES PARALLEL

B. When F_{app} is applied, which TWO choices will be true? (CIRCLE TWO)

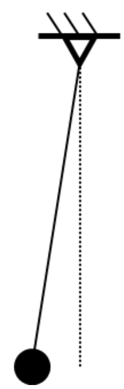
I. $x_1 + x_2 = x_{total}$ II. $x_1 > x_2$ III. $x_1 < x_2$ IV. $x_1 = x_2 = x_{total}$, i.e., x for all parts of the system

V. $F_1 + F_2 = F_{app}$ VI. $F_1 > F_2$ VII. $F_1 < F_2$ VIII. $F_1 = F_2 = F_{app}$, i.e., F for all parts of the system

C. One of your Part B answers should have a (+) symbol. Use Hooke’s law to transform that answer’s equation FROM the *one type of variable that it already has* INTO *the other two types of variables that are in Hooke’s law*.

D. Your other Part B answer should have all (=) signs, i.e., NOT three discrete variables, only one. Substitute this ONE variable into your Part C answer, then simplify/cancel as needed. Show your work.

E. If you had three springs configured as above (with spring constants k_1, k_2, and k_3), what would the equation be to find k_{eff} for the system?



F. A ____________ pendulum is one in which _____ of the mass is considered to be a point mass and is located at the end of the string, i.e., the weight of the ___________ is negligible. In addition, the _________ of displacement of the pendulum must be small, i.e., no greater than 15°.

G. In the figure, label the quantities m, L, x, and θ.

H. Below-right, draw an FBD of the mass when it is at the position shown in the figure.

I. In your FBD, break the mg vector into components // and ⊥ to the bob’s velocity. Label them with what each component would be numerically equal to, taking into account the angle θ. Also, draw axes for reference. Label the axes // and ⊥ .

J. Use Newton’s 2nd law in the // direction. Solve for the acceleration // to the velocity, $a_{//}$.

K. What type of acceleration is your Part J answer? (CIRCLE) CENTRIPETAL TANGENTIAL

L. Based on the definition of the *sin* function, modify your Part J answer to eliminate the θ.

M. Rewrite your Part L answer by placing a (−) sign in front of the right side.

N. If θ is small (< 15°), the directions of the displacement x and your acceleration of Part J are very nearly parallel. With this in mind, why was it okay for us to insert the ‘extra’ (−) sign in Part M? HINT: Think about where $x = 0$.

O. In HW2, P5, we found that, for SHM, $a(t) = -\omega^2\, x(t)$; that is, at any point, $a = -\omega^2\, x$. Substitute this expression into your Part M answer and solve for ω^2.

P. Combine your Part O answer with an equation you wrote in Part A of HW3, P1 to derive a (hopefully) recognizable equation for the period T of a simple pendulum.

Q. Finally, why must it be true that $T \neq mg\cos\theta$ in your FBD and that, in fact, T must be greater than $mg\cos\theta$? HINT: Your Part K answer should somehow point you in the right direction.

Uδ, HW3, P5
Reference Video: "Period of a Physical Pendulum"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

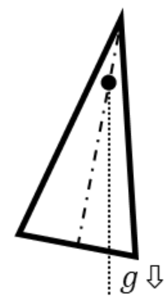
A. A ____________________ pendulum is one in which NOT all of the ________ is assumed to be concentrated at a single point, but rather is distributed in some fashion relative to the pendulum's ________.

B. In HW3, P1, we found that, in SHM, $a = -\omega^2 x$. Write the rotational analog of this equation.

C. Write the Maclaurin series expansion (which, FYI, is a Taylor series centered at x = 0...Who cares?!?) for sin x. Write SIX terms on the equation's right side.

D. Simplify your Part C equation to essentially what it becomes when x is very tiny.

E. Here, we are dealing NOT with x, but with its rotational analog. Rewrite your Part D answer, accounting for this fact.



An object of mass m, uniform density, and rotational inertia I is to oscillate about an axis near the top of the object. The straight-downward direction is indicated by the dotted line, and the object's axis of symmetry is indicated by the dot-and-dash line.

F. Into the figure, draw and label a ⊙ approximately where the object's *com* is located.

G. The distance between an object's *com* and its rotational axis is usually assigned the variable d. Using dimension lines and a two-headed arrow, label this dimension in the figure. Also, use whatever means necessary (arrows or otherwise) to label the displacement angle θ in the figure.

H. Write the equation for the net torque that m's weight exerts, about the axis.

I. Taking into account your Part E answer, rewrite your Part H answer.

J. One translational form of Newton's 2nd law is $F_{net} = ma$. Write the rotational analog of this equation.

K. Substitute your Part I answer into your Part J answer.

L. Now, refer back to the figure, then CIRCLE your answers.

i. When θ is ⤷ from the equilibrium position, the angular acceleration is directed: ⤷ ⤶

ii. When θ is ⤶ from the equilibrium position, the angular acceleration is directed: ⤷ ⤶

M. Ever-so-slightly, modify your Part K answer, in order to account for your Part L responses.

N. Substitute your Part B answer into your Part M answer. Solve for angular frequency.

O. Combine your Part N answer with an equation you wrote in Part A of HW3, P1 to derive an equation for the period T of a physical pendulum.

P. To reiterate, your Part O equation holds ONLY when angular displacements have magnitudes that are very _________.