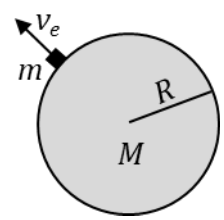
APPC, Mechanics: Unit δ HW 2 Name: ______________________________

Hr: ___ Due at beg of hr on: __________

Uδ, HW2, P1
Reference Videos: (1) "Escape Velocity"
(2) "Binary Star Systems"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

A. **Escape velocity** is the ________________ launching velocity required for an object to ________ ________. (This assumes that no ___________ is added along the way.) To review, all kinetic energies have a sign that is ___, and all absolute gravitational potential energies have a sign that is ___. So, if we don't want to be ________ to Earth, then we need to have a total energy that is at least ______.

In the figure, a mass m is launched with escape velocity v_e from the surface of a celestial body of mass M and radius R. We want a general expression for v_e.



B. Using the variables v_e and R, write an equation depicting the total energy m will have at the instant of lift-off. This is the total energy it will have when it is a distance R from M's center, i.e., $E_{r=R}$ = ?

C. After m is launched at a speed of v_e, its speed thereafter will ________________ with time.

D. Explain your Part C answer. Make reference to one or more aspects of Newton's 2nd law.

E. For the conditions of escape velocity to be satisfied, m must reach a distance of $r = \infty$. Using the variables $v_{r=\infty}$ and $r_{r=\infty}$ (☺), write an equation that depicts the total energy m will have at $r = \infty$, i.e., $E_{r=\infty} = ?$

F. Since we will be conserving energy, use that fact to combine your Parts B and E answers so that relevant terms of the situation are still visible. Do NOT yet simplify.

G. *Mentally* modify the right side of your Part F answer by carrying your Part C answer to its logical conclusion AND by inputting information from the Part E question. Having accounted for these modifications, write your modified Part F equation here.

H. Simplify your Part G answer to obtain an expression for v_e.

I. A **binary star system** consists of two stars that revolve about their common __________ __ ________ (i.e., their ____). We believe that _______ stars in our universe exist in binary star systems.

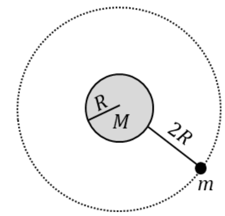
J. Assuming $M > m$, depict (and label) the first sentence of your Part I answer using the figure below. Show the orbital paths (assume circular) for each star and label the radii of the orbits as r_m and r_M.

K. CIRCLE which star has the larger:

i. period of motion	m	M	it's a tie
ii. tangential velocity	m	M	it's a tie
iii. force of gravity on it	m	M	it's a tie
iv. acceleration	m	M	it's a tie

Uδ, HW2, P2

Reference Videos: (1) "A Quick Review of Planetary Motion"
(2) "A Review of Planetary Motion (Satellite Motion) Part II"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist



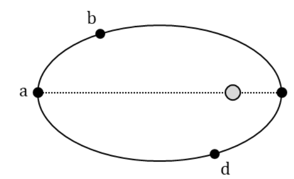
The figure at right shows a celestial body of mass M and radius R being orbited by a satellite m at an altitude of $2R$ above M's surface. Assume m is in circular orbit.

A. Write (and simplify) an expression for the force of gravity on m.

B. Using your Part A answer, apply Newton's 2nd law to derive an expression for the necessary speed v for m to follow a circular path around M.

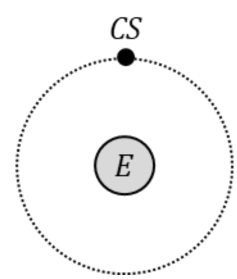
C. Show that your Part B answer is dimensionally correct, i.e., that the right side's units do, in fact, yield m/s.

D. Use your Part B answer in deriving an expression for the angular momentum of m.



E. The next figure shows a satellite in elliptical orbit around a celestial body. CIRCLE all correct answers. At which point(s) in the orbit does the satellite have the...?

i. largest force of gravity:	a	b	c	d	all equal to zero	all equal, but NOT zero
ii. largest torque:	a	b	c	d	all equal to zero	all equal, but NOT zero
iii. smallest angular momentum:	a	b	c	d	all equal to zero	all equal, but NOT zero
iv. largest total energy:	a	b	c	d	all equal to zero	all equal, but NOT zero
v. smallest kinetic energy:	a	b	c	d	all equal to zero	all equal, but NOT zero
vi. largest centripetal acceleration:	a	b	c	d	all equal to zero	all equal, but NOT zero
vii. largest tangential acceleration:	a	b	c	d	all equal to zero	all equal, but NOT zero
viii. largest satellite speed:	a	b	c	d	all equal to zero	all equal, but NOT zero

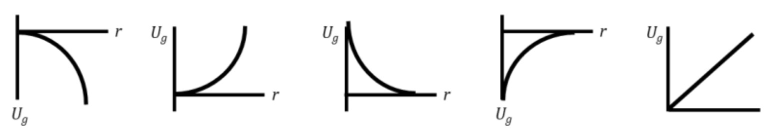


F. The figure shows a communications satellite (CS) in circular orbit around Earth, i.e., the satellite is traveling at the perfect speed for circular orbit. If suddenly the engines on the satellite are fired opposite to the direction of its travel, the satellite will slow down. What simple shape will the new orbit of the satellite have?

G. Based on your Part F answer, draw into the figure an approximate new orbital path.

H. Suppose a planet has a radius equal to 5 Earth radii ($5R_E$) and a mass equal to 125 Earth masses ($125M_E$). (Jupiter has a mass of more than 300 Earth masses, FYI...). In terms of g, determine the gravitational field strength on the surface of this hypothetical planet. Show your work.

I. Which graph exhibits how absolute gravitational potential energy U_g varies with separation r between two masses? CIRCLE your answer.

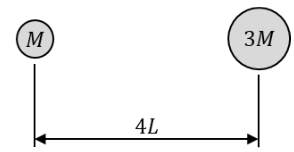


Uδ, HW2, P3

Reference Videos: (1) "A Review of Planetary Motion (Satellite Motion) Part II"
(2) "A Review of Planetary Motion (Satellite Motion) Part III"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

The figure shows a binary star system that will be revolving around a common center of mass (*com*). Assume both stars follow circular orbits.

A. Use an equation to determine where the system's *com* is. Then, into the figure, put a • where the *com* is, label it, and label the distance from the *com* to each star. HINT: Assume Star M is at the origin.



B. Based on your Part A answers, draw into the figure each star's circular path.

C. How should the orbital periods of Stars M and $3M$ compare?

Let's prove your Part C answer. We'll first find an expression for the period of M...

D. Write and simplify the equation for the force of gravity on M.

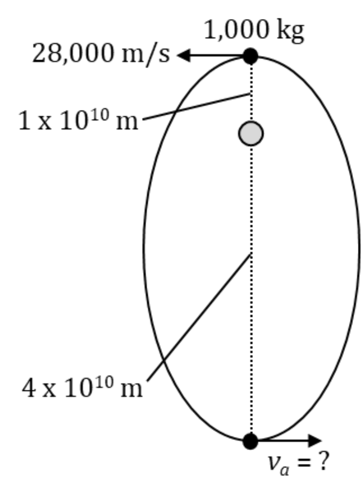
E. Use your Part D answer and Newton's 2nd law to obtain an expression for M's orbital speed v.

F. But speed is distance over time, i.e., $v = \frac{2\pi r}{T}$. Use this equation and your Part E answer to obtain an expression for the period of M.

G. Repeat Parts D-F below, but this time for the $3M$ star.

H. Were you able to verify your Part C answer? (CIRCLE) 😊 😩

The figure at right shows an asteroid in an elliptical orbit around a star. Assume that the asteroid's mass remains constant.



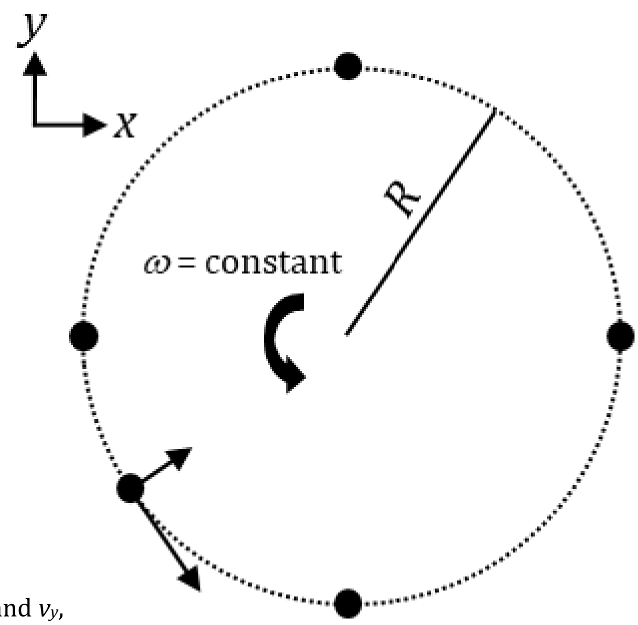
I. For the two points shown in the figure, conservation of angular momentum boils down to the simple equation shown back in Part M of HW1, P5. Use that equation and the information in the figure to determine the speed v_a.

J. Assuming the mass of the star is 7.346×10^{28} kg, now use the conservation of mechanical energy to determine v_a.

K. How do your Parts I and J answers compare?

Uδ, HW2, P4

Reference Video: "Kinematics of Simple Harmonic Motion (Part I)" YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



<u>NOTE</u>*: The line of reasoning here and in subsequent assignments differs slightly from the videos. Nonetheless, you will benefit from having watched those in advance of trying the next few assignments.*

The figure shows an object in uniform circular motion (UCM). PLEASE IMAGINE that the *xy*-axes shown are at the circle's center.

A. Make the following additions to the figure.

i. One point on the circle already has two vectors. Label these (correctly!) as $\vec{v}$ and $\vec{a}$.

ii. At the other four designated points, draw and label $\vec{v}$ and $\vec{a}$ vectors of the correct direction and length.

iii. On the pre-drawn $\vec{v}$ vector (see Part Ai), draw components v_x and v_y, based on the *xy*-axes shown. (Again, imagine that these are at the center).

iv. On the pre-drawn radius *R*, draw in and label its components as *x* and *y*.

v. Label the angle between *R* and *x* as θ.

B. Let's digress for a minute... Write the:

i. velocity bridge equation

ii. acceleration bridge equation

C. Write our often-used equation for centripetal acceleration.

D. Substitute the right side of your Part Bi answer into your Part C answer and then simplify to obtain a (for us) lesser-used equation for centripetal acceleration.

E. One last thing: Write what *x* is equal to, in terms of *R* and θ. (Look again at the figure.)

F. Apply the next steps to ONLY the figure's four 'compass points', i.e., NOT to the point with the pre-drawn $\vec{v}$ and $\vec{a}$ vectors.

i. Put the appropriate subscript, either $_x$ or $_y$, onto those points' $\vec{v}$ and $\vec{a}$ labels.

ii. Enclose each of the $\vec{v}$ and $\vec{a}$ labels in absolute value signs and then write " $= max$ " next to each one. Verify mentally that this claim is true for each of the compass-point $\vec{v}$ and $\vec{a}$ vectors.

iii. TWO of these four [$a_x = 0$ $a_y = 0$ $v_x = 0$ $v_y = 0$] apply to each compass point. Write the correct two of them near each point.

iv. ONE of these two [$|x| = max$ $x = 0$] applies to each compass point. Write the correct one near each point.

G. CIRCLE every quantity in the figure that refers to the *x*-direction.

Let's digress again... Since $velocity = \frac{displacement}{time}$, we recall that $\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t}$.
If we rename θ_f as just θ and θ_i as ϕ, we get... $\omega = \frac{\theta - \varphi}{t}$.

H. Solve this last expression for θ.

I. Substitute your Part H answer into your Part E answer.

J. Your Part I answer is an equation for *x*-position vs. time. If you differentiate this once, you'll get an equation for the *x*-____________ vs. time; if you differentiate it once more, you'll get an equation for the *x*-_______________ vs. time.

K. Differentiate your Part I answer:

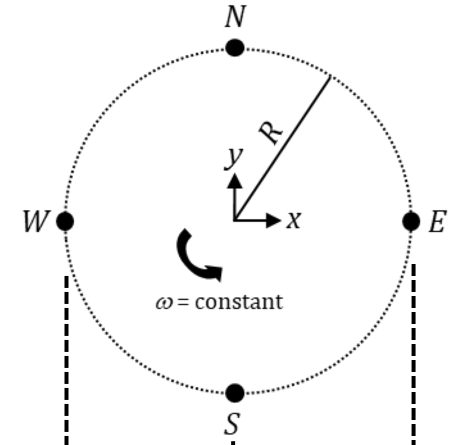
i. ...once here...

ii. ...and once more over here.

(Your answers will agree with the video, with the notable exceptions that you'll have *R* instead of *A* and ($\omega t + \phi$) instead of (ωt).

Uδ, HW2, P5

Reference Videos: (1) "Kinematics of Simple Harmonic Motion (Part I)"
(2) "Kinematics of Simple Harmonic Motion (Part II)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



We continue from HW2, P4...

A. The maximum value that a *sin* or *cos* function can have is ____.

B. Given your Part A answer, your Parts I and K answers from P4 reveal that, for the circular motion depicted at the top of that assignment and partially reproduced in the figure at right...

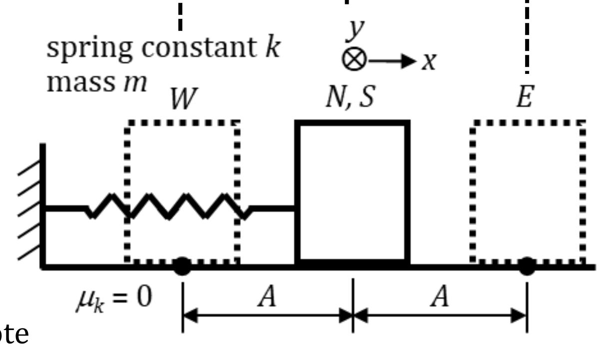
i. $|x_{max}| =$ ____ ...which occurs at the following TWO points: N S E W

ii. $|v_{x,max}| =$ ______ ...which occurs at the following TWO points: N S E W (CIRCLE)

iii. $|a_{x,max}| =$ ______ ...which occurs at the following TWO points: N S E W

C. State how what you wrote in the blanks in Parts Bii and Biii above compares to Parts Bi and D from P4.

D. State how what you circled in Part B compares to the MAX *x*-quantities you circled in Part G on P4.



The thinking and mathematical formulating we've done so far in P4 and P5 (specifically, in the *x*-direction) correspond EXACTLY to the kinematics of a mass *m* oscillating on a spring of constant *k*. The figure at right shows such a case, on a frictionless surface. It is as if the figure at the top of the page is a 'bird's eye' view and the figure here is an elevation view. Note here how the same *xy*-axes are designated. Note also the 'drop-down' lines for points N, S, E, and W.

E. Write THREE of the following six choices onto EACH depiction of the mass in the figure.

$|x| = max$ $\quad x = 0 \quad$ $|v_x| = max$ $\quad v_x = 0 \quad$ $|a_x| = max$ $\quad a_x = 0$

F. State how your Part E answers compare to your Part B answers.

G. Look at both figures: What is the mathematical relationship between *A* and *R*?

H. State how what you wrote in the blank in Part Bi compares to your Part G answer.

I. Write three equations by substituting your Part G answer into your Parts I and K answers from P4.

$x(t) =$ $\qquad$ $v(t) =$ $\qquad$ $a(t) =$

** *Your Part I answers are the kinematics equations for simple harmonic motion (SHM) of a mass on a spring. Hopefully, now, you see the correspondence between the SHM of a mass on a spring and UCM.*

J. The ϕ term in your Part I answers has to do with WHERE *m* starts on its back-and-forth journey, i.e., when *t* = 0. (More on this later.) The graphs below depict position-vs.-time for *m* starting at various points. Your task here is to (1) figure out how the graphs correspond to the two figures above, and (2) write one letter – either N, S, E, or W – at EACH point of maximum *x* AND EACH point where *x* = 0.