APPC, Mechanics: Unit δ HW 1

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

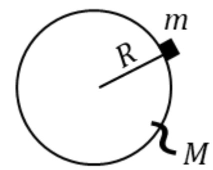
Uδ, HW1, P1
Reference Video: "Gravitational Field Strength or Acceleration Due to Gravity"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

A. Write the simple vector equation for gravitational field strength.

B. Up to this point in the course, what name have we used for the quantity on the left side of your Part A answer, AND what unit have we always used for it?

C. From the right side of your Part A answer, gravitational field strength also has what other unit?

D. Show that the units from your Parts B and C answers are equivalent.



The figure shows a small mass *m* at the surface of a planet of mass *M* and radius *R*.

E. Insert the smaller mass *m* into your Part A answer and rearrange the equation into a NO-denominators form.

F. If needed, refer WAY back to Part A of Uα, HW3, P2 (our first unit of study) and write the expression for the force of gravity F_g between *M* and *m*. Leave out the (–) sign, for now.

G. Combine your Parts E and F answers and simplify to obtain an expression for *g* at the surface of any body of mass *M* and radius *R*.

H. Rewrite your Part G answer as a vector equation.

I. There should be a unit vector in your Part H answer. Write that unit vector again here: _____

i. Why is this unit vector designated with the specific letter that it is?

ii. In which direction does this unit vector always point?

J. What is the meaning of the (–) sign in your Part H answer? (In your response, feel free to mention your Part Iii answer.)

K. In terms of geometry, we will assume that all spinning planets have the 3-D shape of a perfect ______________. In reality, spinning planets (e.g., Earth) are ______________ __________________.

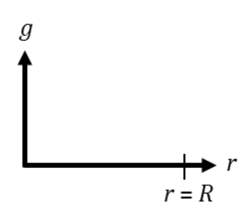
L. Write the basic equation for density, in BOTH words AND variables.

M. Keeping in mind that we are dealing with the situation shown in the figure above, substitute the equation for the volume of your *first* Part K answer into your Part L answer, and then solve for *M*.

N. Substitute your Part M answer into your Part G answer, and solve for *g*.

O. Rewrite your Part N answer using *r* in place of *R*.

P. Your Part O answer applies for finding *g* anywhere WITHIN a spherical planet of uniform density *D*, i.e., it applies for all $r \le R$. At right, sketch a simple graph, showing how *g* varies with *r*, inside such a planet. (This IS easy; just look at your Part O equation.)

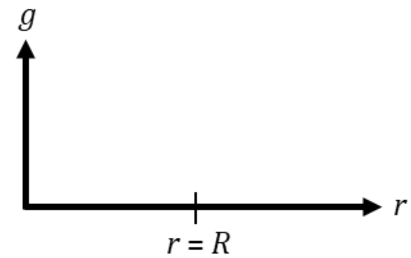


Uδ, HW1, P2

Reference Videos: (1) “Gravitational Field Strength or Acceleration Due to Gravity”
(2) “Gravitational Fields”
(3) “Gravitational Potential Energy”
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

First, let’s finish some earlier work. When the last assignment ended, we were dealing with how g varies with r WITHIN a spherical planet of uniform density D. What about beyond that planet’s surface? Okay...

A. Into the graph at right, recopy your Part P answer from HW1, P1.
NOTE: This extends ONLY to the point where $r = R$; don’t go past that.



B. Rewrite your Part G answer from the previous assignment.

C. Rewrite your Part B answer above using r in place of R.

D. Your Part C answer is the one needed for finding g anywhere AT OR BEYOND the surface of a spherical planet of uniform density D, i.e., it applies for all $r \geq R$. Into the graph above, sketch how g varies for $r \geq R$, according to your Part C equation. The graph will have two distinct parts – i.e., Part P previously, and Part C now – that have CLEARLY different shapes. Above each of the two parts of the graph, indicate how the g field relates to r, i.e., you need to write (TWICE!) ... $\boxed{g \propto ?}$ ...where the only variable in the ? is some variation of r, such as…r, r^2, $\frac{1}{r}$, $\frac{1}{r^2}$, etc.

***<u>To be clear</u>**: While the Part P equation CANNOT be used for $r \geq R$, there IS a modification that can be made to the Part C equation so that it WILL work for $r \leq R$... but let’s NOT go there now.*

E. Write the vector equation for the force of gravity between two masses m_1 and m_2, whose centers are separated by a distance r. HINT: Your answer should have one vector symbol, one (–) sign, and one unit vector.

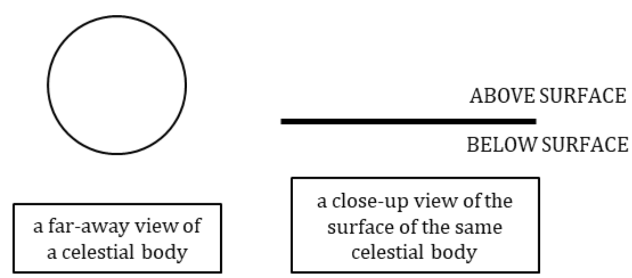
F. The unit vector ____ that appears in your Part E answer always points ________________ ________ ________

the object on which you are considering...

G. Below-right, draw how gravitational field lines appear in the vicinity of each of the two figures. HINT: Remember that a gravitational field is a *vector* field.

H. Somewhere into the figures at right, write either...

($g \approx constant$) OR ($g \neq constant$)



I. Write the value and units for the universal gravitational constant, a.k.a., the Cavendish constant.

J. Write equations for: i. change in gravitational potential energy near a planet’s surface

ii. absolute gravitational potential energy in a two-mass system

K. For EACH of your Part J answers, show that a force of gravity equation is, in fact, the negative-derivative-with-respect-to-position (basically, distance) of a gravitational potential energy equation.

Uδ, HW1, P3

Reference Videos: (1) "Gravitational Potential Energy"
(2) "VID00012 (Gravitational Forces and Grav. Potential Energy)"
YouTube, lasseviren1

A. Any time a mass moves AGAINST a gravitational field, it _________ gravitational potential energy; any time a mass moves WITH a gravitational field, it _________ gravitational potential energy.

B. CIRCLE the correct answers.

i. Absolute gravitational potential energies: ARE ALWAYS (+) ARE ALWAYS (–) CAN BE (+) or (–)

ii. Changes in gravitational potential energies: ARE ALWAYS (+) ARE ALWAYS (–) CAN BE (+) or (–)

C. A 68 kg person stands on Earth's surface. Earth has a mass of 6.0×10^{24} kg and a radius of 6.4×10^{6} m. Determine the absolute gravitational potential energy stored in the person-Earth system.

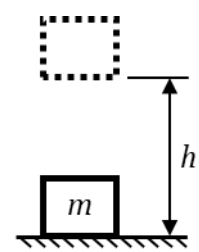
D. Be sure that your Parts Bi and C answers are in agreement. Once they are, draw a smiley-face : _____
Now, with regard to the person in Part C...What does it mean, for the person, that the potential energy you calculated in Part C conforms to your Part Bi answer?

E. How much kinetic energy would the person need to have, in order to NOT be limited by your Part D answer?

F. Determine the speed the person would have to attain, in order to NOT be limited to your Part D answer.

We now show that, when objects are near a planet's surface and undergo elevation changes that are a small fraction of the planet's radius, the easy gravitational potential energy equation from your first-year class ($\Delta U_g = mgh$) is valid.

Consider the figure at right. The mass m is near the surface of a planet having mass M and radius R. The mass starts on the surface and is then elevated to a height h above the surface.



G. Using absolute gravitational potential energy, write equations for when the mass is:

i. on the surface: $U_{g,\,initial} =$ ii. at the elevation h: $U_{g,\,final} =$

H. Use your Parts Gi and Gii answers and obtain a "first-draft" expression for ΔU_g. By "first-draft," we mean that your expression is completely UNsimplified, i.e., it comes straight from the definition of the symbol Δ.

I. Somewhat-simplify your Part H answer by factoring out what you can. Your answer should have two fractions within a single set of parentheses. Also, make sure your answer has only ONE (–) sign.

J. Get a common denominator for the fractions inside the ( ) of your Part I answer and then simplify (only) the numerator. Your answer should have NO (–) signs and a set of ( ) in the denominator.

K. Now, given that $R >>> h$, the $(R + h)$ in your Part J answer is basically equal to ___. Go ahead and substitute this into your Part J answer and write the result here.

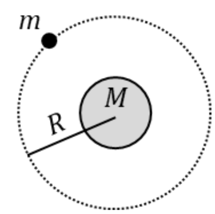
L. Look back at your Part G answer from HW1, P1 and substitute this into your Part K answer. *Voila!* Sure enough, when $h <<< R$, finding ΔU_g is a piece of cake. ☺

Uδ, HW1, P4

Reference Videos: (1) "Satellites in Circular Orbit"
(2) "Kepler's Laws of Planetary Motion"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

A. In physics, a **satellite**: (CIRCLE)

a. could be a human-made object
b. could be a Nature-made object
c. could be either a human-made object OR a Nature-made object
d. is a lamp cowboys use to find where to sit on their horses when it's dark



B. The figure shows a satellite *m* in a circular orbit of radius *R* around a planet of mass *M*. Into the figure, draw and label a gravitational force vector on the satellite.

C. The satellite is undergoing uniform circular motion (UCM). It therefore has NO ______________________ component of acceleration; it has only ______________________ acceleration. For UCM, this last component of acceleration conforms to the easy equation...

D. There is no friction in space, so the force vector you drew in Part B is the only one on the satellite. Write a Newton's 2nd law equation for the satellite, substituting into it the last of your Part C answers.

E. Use Newton's law of gravity to write another equation for the force of gravity on the satellite. (See Part F of HW1, P1 and – again – leave the (–) sign out.)

F. Combine your Parts D and E answers and solve for the speed required for this satellite to maintain a circular orbit around the planet.

G. Write the integral-and-dot-product form of the work equation. (Don't forget vector signs.)

H. Into the figure above, draw and label a correct $\overrightarrow{dx}$ vector (or $\overrightarrow{ds}$, it doesn't matter) on the satellite.

I. For a satellite in a circular orbit, the work done on the satellite by the force of gravity is...

J. Explain your Part I answer. In your response, make reference in some way to your Part G answer.

K. From your Part I answer, we know that, over time, the speed of the satellite will...

L. What fundamental law or principle of physics led you to your Part K answer?

J. Kepler's first law says that planets travel around the Sun in ____________________ paths, with the ______ at one focus. The second law says that a given planet will sweep out equal ________ in equal ________. The third law says that the ratio of a planet's ________________ __________ squared versus its ________________ __________ cubed is a constant, which has the same numerical value for ____ planets in the solar system.

K. Kepler's second law is derived directly from WHICH fundamental law or principle of physics?

L. Write an equation that expresses the last sentence of your Part J answer. (There are several acceptable responses to this Part.)

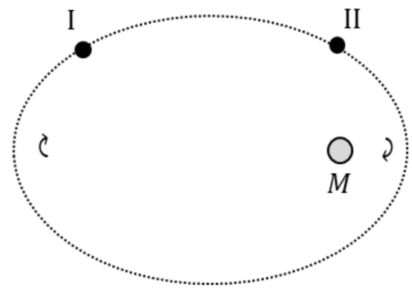
M. In Part J, you answered that the *R* term was called the planet's ________________ __________. More precisely, it is the length of the _______-_________ _______, which is equal to the fraction of _____-_______ of the "big" dimension of the ____________________ path taken by the planet.

Uδ, HW1, P5

Reference Videos: (1) "Elliptical Orbits and the Conservation of Energy"
(2) "Elliptical Orbits and Conservation of Angular Momentum"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

A. What condition exists for satellites – for both elliptical and circular orbits – that results in mechanical energy always being conserved?

A satellite of mass *m* is in elliptical orbit around a celestial body (planet or star) of mass *M*. Two points on *m*'s orbit are shown, labeled I and II.



B. State one specific piece of information about *M*'s location, within the ellipse. (HINT: Your math teacher would be pleased that you know enough to say this.)

C. Into the figure, draw and label the position vectors $\vec{r}_I$ and $\vec{r}_{II}$.

D. In the correct direction AND to an appropriate length, draw velocity vectors $\vec{v}_I$ and $\vec{v}_{II}$ into the figure.

E. To review, write basic equations for: i. kinetic energy ii. absolute gravitational potential energy

F. Use your Parts C-E answers to match each term in the left column to a term in the right column.

K_I	SMALL AND (+)
K_{II}	SMALL AND (–)
$U_{grav,\,I}$	LARGE AND (+)
$U_{grav,\,II}$	LARGE AND (–)

G. Write an equation illustrating the conservation of mechanical energy for the satellite with regard to Points I and II. Simplify your equation only to the extent of cancelling any factors common to all terms.

H. Assume we know r_I, r_{II}, v_I, and v_{II} (and *G*, obviously). Solve your Part G answer for the unknown mass of the celestial body *M*. HINTS: Simplify your expression to the point where you have, in the denominator, two fractions within a single set of ( ). Also, your answer should have TWO (–) signs.

I. What condition is true whenever angular momentum is conserved?

J. Into the figure, as best you can, draw and label gravitational force vectors $\vec{F}_{g,I}$ and $\vec{F}_{g,II}$ that act on the satellite. The vectors should be in the correct direction AND have an appropriate length.

K. Making reference to your Part J answer, explain how your Part I answer is true for the satellite above.

L. Assuming *M* is the Sun, label the points of **aphelion** and **perihelion** in the figure. If *M* were Earth, those points would be the **apogee** and **perigee**. Write these terms into the figure as well, in a sensible place.

M. For the terms in Part L, conservation of angular momentum boils down to the very easy $r_A v_A = r_P v_P$. Why is a similarly simple equation NOT applicable for any other points, including I and II in the figure?