APPC, Mechanics: Unit β HW 4

Name: _____

Hr: _____   Due at beg of hr on: _____

Uβ, HW4, P1
Reference Video: "Work Done by a Conservative Force"
                YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

A. Write the most basic equation involving the work done by a conservative force and the change in potential energy. HINT: Your answer should have only one 'equals' sign.

B. With reference to the figure, corroborate your Part A answer by determining the:

   i. work done by gravity in raising the mass

   ii. change in potential energy when the mass is raised

   iii. work done by gravity in lowering the mass

   iv. change in potential energy when the mass is lowered
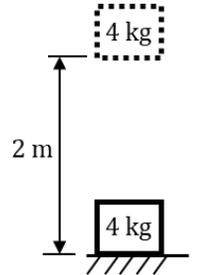
C. From the video, write the equation relating a conservative force, work done by that force, the change in potential energy, and displacement. HINTS: You need two (=) signs, one integral, one differential, and the dot product.

D. Differentiate your entire Part C answer to obtain an equally-applicable equation. HINTS: You'll need two (=) signs, three differentials, and the dot product.

E. If you ignore the 'work' part of your Part D answer and rearrange the rest, you will see that a conservative force is the _____ derivative of the _____ _____ with respect to position. Thus, on a graph of _____ _____ versus position, the force at any particular position is found to be the _____ _____ of the graph at that position.

F. Refer to the figure at right. Circle ALL correct answers.

   i. *F* is large and (+) at point(s)...............................a b c d e f g h i

   ii. *F* is equal to zero at point(s)...........................a b c d e f g h i

   ii. *F* is moderate and (–) at point(s)......................a b c d e f g h i

   iv. Point(s) of **stable equilibrium** is/are............a b c d e f g h i

   v. *F* is moderate and (+) at point(s).....................a b c d e f g h i

   vi. *F* is large and (–) at point(s).............................a b c d e f g h i

   vii. Point(s) of **unstable equilibrium** is/are......a b c d e f g h i
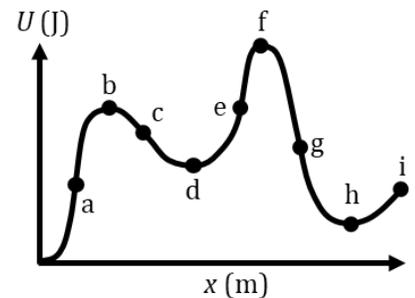
G. Let's look at your Parts Fiv and Fvii answers. Since our conservative force and displacements are acting in only 1-D, let's take → to be (+) and ← to be (–). In the unshaded portions of the table, circle your answers.

Which points?	If we scoot one of them a little bit...	...the graph's slope becomes...	...which means the force will be...	...and will have the direction...	...which DOES WHAT to the original scoot?
Part Fiv points	→	+ –	+ –	← →	Undoes it. Adds to it.
Part Fiv points	←	+ –	+ –	← →	Undoes it. Adds to it.
Part Fvii points	→	+ –	+ –	← →	Undoes it. Adds to it.
Part Fvii points	←	+ –	+ –	← →	Undoes it. Adds to it.

Uβ, HW4, P2
Reference Video: "Review of Work, Energy, and Power (Part I)"
                    YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

A. Determine the work done when a force $\vec{F} = -2\,N\,\hat{\imath} + 5\,N\,\hat{\jmath} - 1\,N\,\hat{k}$
   acts over a displacment $\vec{s} = -5\,m\,\hat{\imath} - 2\,m\,\hat{\jmath} + 3\,m\,\hat{k}$.

B. Based on the sign of your Part A answer, is the angle between the vectors **acute** or **obtuse**?

C. Determine the magnitude of: i. $\vec{F}$

                                      ii. $\vec{s}$

D. i. Use your Parts A and C answers to determine the angle
      between the vectors $\vec{F}$ and $\vec{s}$. Refer back to Part A of
      HW3, P2, if you've forgotten the equation that applies.

   ii. Does your Part Di answer corroborate your Part B answer?

Use the graph to answer the next two questions.

E. Determine the work done between $x = 0$ m and $x = 6$ m.

F. If the object's initial speed is 4 m/s, determine its speed at $x = 6$ m.

F_x (N)

$m = 8$ kg
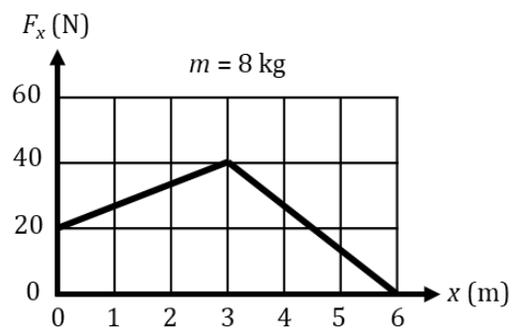
G. The figure shows a spring launching a mass from rest. The mass slides along the frictionless surface.
   Determine the mass's speed at the top of the hill.

$v = ?$

$k = 240$ N/m
$x = 2$ m

$m = 10$ kg

2 m $\mu_k = 0$ 5 m
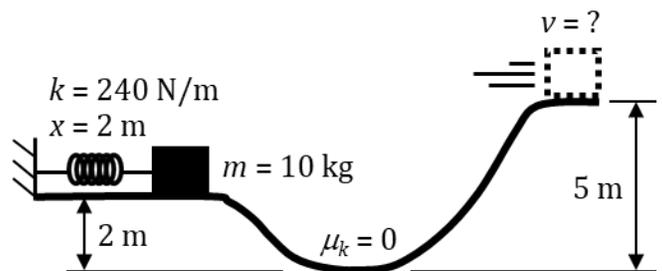
H. In the last figure, the scenario is repeated, except now there IS friction and the mass's speed at the top
   of the hill is measured to be 4 m/s. Determine the work done by friction.

$v = 4$ m/s

$k = 240$ N/m
$x = 2$ m

$m = 10$ kg

2 m $\mu_k \neq 0$ 5 m
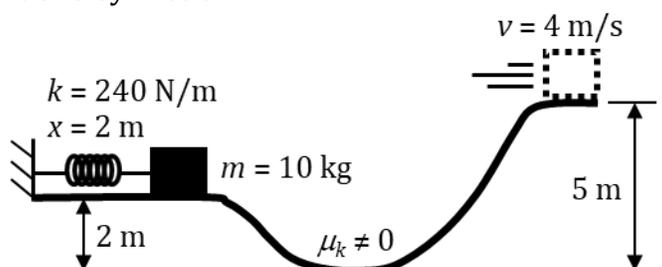
Uβ, HW4, P3
Reference Video: "Review of Work, Energy, and Power (Part II)"
                   YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

3 kg
$v_o = 0$ m/s
6.4 m
1 kg
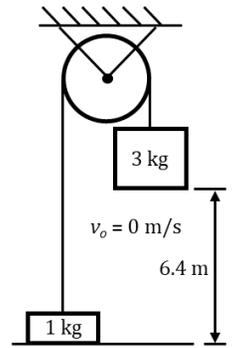
A. Determine the speed with which the 3 kg mass hits the floor.

B. A 2 kg mass moves in accord with the velocity-time function $v(t) = 6t^2 - 6t - 5$.
   At time $t = 0$ s, the mass is at position $x = 0$ m. Determine the:

   i. initial velocity

   ii. position at $t = 3$ s

   iii. net force at $t = 3$ s

   iv. work done on the mass between $t = 0$ and $t = 3$ s

   v. power being expended at $t = 3$ s

U (J)
$m = 5$ kg
60
40
20
0
0 1 2 3 4 5 6
x (m)

Based on the graph at right, answer the following questions.
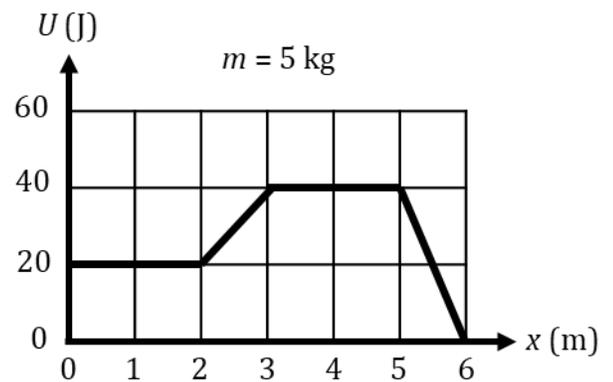
C. Determine the net force on the mass at:

   i. $x = 1.5$ m   iii. $x = 3.5$ m

   ii. $x = 2.5$ m   iv. $x = 5.5$ m

D. At what specific time (OR time interval, whichever
   applies here) must the mass have its greatest speed?

E. Explain briefly your Part D answer.

F. Suppose at this point that you are told that, at some specific
   time (OR time interval, whichever applies here), the mass
   is at rest. This being the case, determine the value of the
   maximum speed that relates to your Part D answer.

Uβ, HW4, P4
Reference Video: "Review of Work, Energy, and Power (Part III)"
                   YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

A. Determine the following quantities. Assume g = 10 m/s² and that air
   resistance is negligible. You will NOT need a calculator for Part A.
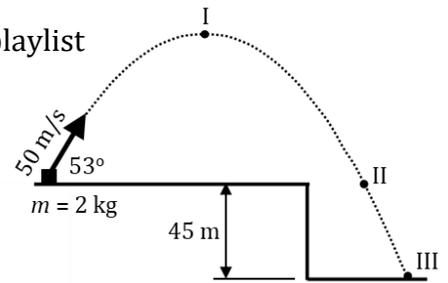
   i. time to reach I

   ii. velocity at I

   iii. height above launch point, at I

   iv. time to reach II

   v. velocity at II

   vi. time to reach III

   vii. horizontal range, from launch to III

   viii. work done by gravity between launch and III

B. Determine the mass's kinetic energy at launch <u>AND</u> at III.

C. Show that your answers to Parts Aviii and B
   conform to the work-energy theorem.

D. A potential energy vs. position function is $U(x) = -3x^2 + 4x - 2$. The object is a 2 kg mass.

   i. Determine the force on the mass AND the mass's acceleration when it is at the position x = 2 m.

   ii. Determine the position at which
       there is zero force on the mass.

   iii. Is your Part Dii answer a point of *stable* or *unstable* equilibrium? Show justification for your claim.
        HINT: You need to figure out what the $U(x)$ function is doing on either side of your Part Dii answer.

E. For the situation where the graph applies, it has been determined that the system's total energy is 40 J.

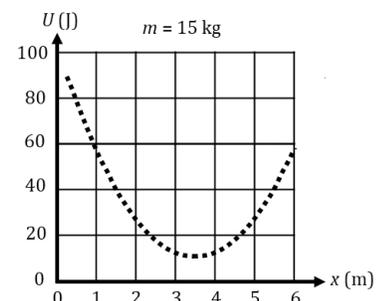   i. State why the mass CANNOT reach the position of x = 1 m.

   ii. The two boundary x-values for the actual motion of the mass are

       the lower bound of x = ____ m and the upper bound of x = ____ m.

   iii. (CIRCLE) In which direction does the force act on the mass...

       a. at the lower bound ← →   b. at the upper bound ← →

   iv. The mass is released from rest at x = 1.5 m. Find v_{max} AND the position where v_{max} occurs.

Uβ, HW4, P5
Reference Video: "Review of Work, Energy, and Power (Part IV)"
YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

In the figure at right, a mass is released from rest at the top of a slope. Initially, the slope is frictionless; friction begins to act only when the mass has fallen halfway down to its final elevation. The mass eventually comes to rest at the point where it is shown as a dashed object.
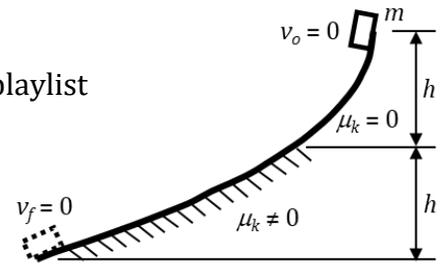
A. Derive an expression for the speed of the mass after it has fallen a vertical distance h.

B. Why does the normal force do zero work, in bringing the mass to rest?

C. For the entire trip, how much work does gravity do on the mass?

D. For the entire trip, how much work does friction do on the mass?

E. Justify your Part D answer. In your response, make reference to the work-energy theorem.

F. A force vs. position function is $F_{net}(x) = 8x^3 - 3x^2 - 4x + 1$. The object is a 4 kg mass.

i. Determine the mass's acceleration at position $x = 1$ m.

ii. Determine the work done by the force between $x = 0$ m and $x = 2$ m.

iii. If the mass had $v_o = 0$ m/s at $x = 0$ m, determine its speed at $x = 2$ m.

G. A mass m is attached to a rod of negligible mass and length R. The rod is initially horizontal and the system is at rest. The mass is dropped and the rod rotates about its pinned left end. Derive an expression for the speed of the mass at the moment when the rod passes through the 30° angle shown in the figure.
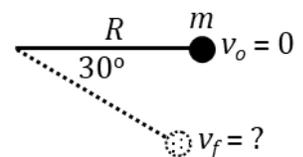
H. A 2 kg mass is launched upward, with $v_o = 10$ m/s. It reaches a height of 4 m. Air resistance is NOT negligible. Between launch and the top, determine the following:

i. the net work done on the mass

ii. the work done by gravity

iii. the work done by the friction of air resistance