APPC, Mechanics: Unit β HW 3

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

Uβ, HW3, P1

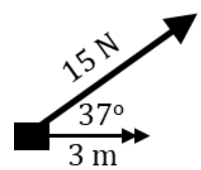
Reference Videos: (1) "The Scalar Product or Dot Product for Physics"
(2) "Dot Product"
YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

A. Why is it called the… i. …**scalar** product?

ii. …**dot** product?

B. Up until this point, we have had lots of practice **adding** vectors; now we will be __________________ vectors. There are _____ different ways we can do this. We use the **dot product** (or **scalar product**) when we find we need to use two vectors that are oriented _______________ to each other. We use the **cross product** (or **vector product**) when we need to use two vectors that are oriented ______________________ to each other. (More on the cross product at a later time. ☺) One very important physical quantity that requires our use of the dot product is ______, and the two vectors (or vector components) that must be oriented _______________ to each other to calculate this are ________ and ______________________. Again, when we use the dot product, our result is always a ____________ quantity; it will always have a ________________ and will generally have a ______, but it will never have a _____________.

C. Find the work done by the force shown in the figure. You can see that the full force and the full displacement are NOT in quite the same direction. (Note also that the narrator uses $\vec{s}$ for displacement, but feel free to use whatever variable symbol makes you comfortable.) Do NOT use a calculator on this problem.



D. In general, for any two vectors $\vec{A}$ and $\vec{B}$, the magnitude of the dot product is found by using the equation…

E. Now, use a calculator to verify that your Part D answer, applied to Part C, does – in fact – give you the same answer as what you already showed in Part C. Show your work at right.

F. For taking the dot product of two vectors in unit-vector form, e.g., taking $\vec{A} \cdot \vec{B}$, where $\vec{A} = a_x\hat{\imath} + a_y\hat{\jmath}$ and $\vec{B} = b_x\hat{\imath} + b_y\hat{\jmath}$, we DO need to (here) "FOIL" our terms, just like we always do with polynomials. BUT…Why, then, is the net result merely $\vec{A} \cdot \vec{B} = a_x b_x + a_y b_y$? (i.e., Where did $a_x b_y$ and $a_y b_x$ go?)

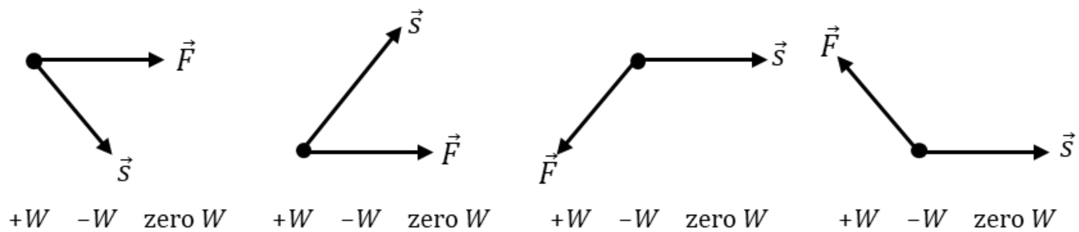
G. Determine the work done when a force $\vec{F} = 3\,N\,\hat{\imath} + 2\,N\,\hat{\jmath} + 1\,N\,\hat{k}$ acts over a displacment $\vec{s} = 2\,m\,\hat{\imath} + 7\,m\,\hat{\jmath} + 5\,m\,\hat{k}$.

Uβ, HW3, P2

Reference Video: “Work Done on an Object by a Constant Force”
YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

A. Write the equation from the video for finding the work done by a constant force $\vec{F}$ if the displacement is $\vec{s}$. HINTS: The equation should have TWO equals signs. Also, put vector symbols over vector quantities, and ONLY over vector quantities.

B. Circle the type of work for each case at right: (+) work, (–) work, or zero work.



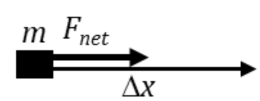
C. i. Zero work is done whenever…

ii. This is the case whenever the motion of the object is…

iii. Briefly explain how your answers to Parts Ci and Cii connect to your Part A answer. In your response, specifically mention the angle θ, i.e., what θ IS and how its presence in the equation connects to your two previous answers.

D. From the video, write the simplest equation for the **work-energy theorem**.

We will now derive a more specific and useful form of this theorem. Consider the figure at right, which shows a constant net force acting through a displacement.



E. Write the expression for the net work done W_{net} by the force. (This is very easy, since F_{net} and Δx are in exactly the same direction. ☺)

F. Write the Newton’s 2nd law equation that applies here.

G. Substitute your Part F answer into your Part E answer.

H. Divide both sides of your Part G answer by the mass.

I. Write the kinematics equation sometimes called “The No-Time Equation.” If you need to look this up, one place it can be found is on the assignment Uα, HW1, P3.

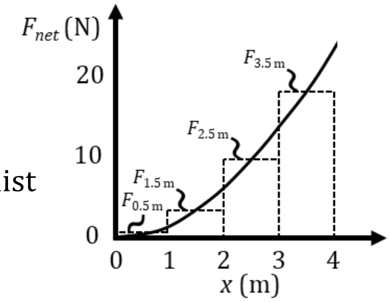
J. Substitute your Part H answer into your Part I answer.

K. Solve your Part J answer for W_{net}. (*Voila*! A useful work-energy theorem equation!)

L. Back in Part C, we were dealing with cases in which zero work is done. Your Part K answer shows that, in cases of zero work being done, the speed of the object will…

Uβ, HW3, P3

Reference Videos: (1) “The Work Done by a Varying Force”
(2) “Work Done on an Object by a Varying Force”
YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist



Until now, any time we’ve dealt with work, we’ve assumed that the force is constant. Here, we address the case of work done by a force that is changing.

Above-right is a graph showing how a force changes smoothly and continuously with displacement. The graph conforms to the equation $F(x) = \frac{3}{2}x^2$. Several rectangles are shown; we’ll get to those in a minute.

A. Suppose we want to know the work done by the force between x = 0 m and x = 4 m.
What feature of an F-x (or F-s, whatever…) graph will give us this quantity?

B. One way to approximate your Part A answer is divide the graph into rectangles. Here, see that each rectangle’s height is the force at the “x.5” m displacements. What is the width of each rectangle?

C. Write out the equation, with ALL terms, that approximates what you stated in your Part A answer. Use given heights (i.e., the variables) from the graph as well as your Part B answer. Label the left side of the equation according to the physical quantity you are calculating.
HINT: You will need three (+) signs.

D. What about the rectangles shown will result in your Part C answer yielding a good approximation to your Part A answer? HINT: Look at the graph’s curve and how the rectangles ‘interact’ with it.

E. Recall that, here, the actual equation of the graph is $F(x) = \frac{3}{2}x^2$. Now, use a calculator to evaluate your Part C answer. Do NOT round, for any calculation you make. Include the correct unit on your answer.

F. In approximating our Part A answer using the method of rectangles, what would happen to our accuracy if we were to use more – but narrower – rectangles?

G. Our ACTUAL Part A answer would be found if we used HOW MANY rectangles?

H. State the thickness/width that each Part G rectangle would need to have.

I. Your Parts G and H answers are most easily accomplished by doing what, to $F(x)$?

J. Carry out your Part I answer here. Recall that we want to know the work done by the force between x = 0 m and x = 4 m.

K. Comment on how your Parts E and J answers compare.

L. Now, determine the work done by the force between x = 2 m and x = 4 m.

M. In Part J, the displacement is exactly twice what it is in Part L. Why, then, ISN’T your Part J answer exactly twice your Part L answer?

Uβ, HW3, P4

Reference Video: “Power and Efficiency”
YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

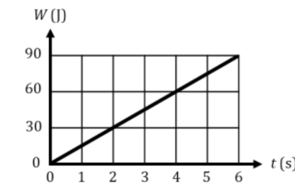
A. Use English WORDS to write two “equations” for average power.

$P_{avg} =$ ____________ $P_{avg} =$ ____________

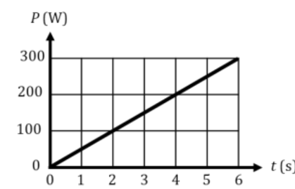
B. Use calculus-type variables to write two equations for instantaneous power.

$P_{inst} =$ ______ $P_{inst} =$ ______

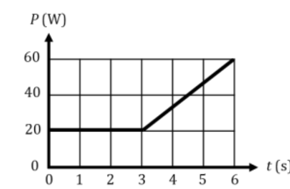
C. Use the graph at right in conjunction with your answers above to determine the power being expended at any instant between $t = 0$ s and $t = 6$ s.



D. Use the graph at right to find the work done between $t = 1$ s and $t = 4$ s.

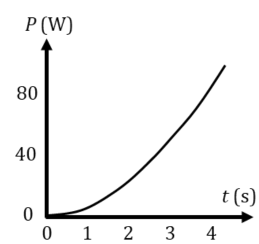


E. Use the graph at right to find the work done between $t = 2$ s and $t = 5$ s.



F. The graph shown here applies to the function $P(t) = 6t^2 - 4t + 5$. Find the work done between:

i. $t = 0$ s and $t = 2$ s ii. $t = 0$ s and $t = 3$ s iii. $t = 2$ s and $t = 3$ s



G. With regard to your Part F answers…You perhaps can see that, if you want to find the area under a function between a lower bound of $x = A$ and an upper bound of $x = B$, in effect what you are doing is finding the area between $x =$ _____ (fill this blank in!) and $x = A$, then finding the area between $x =$ _____ (this blank, too!) and $x = B$, and then…doing what?

H. Write the two equations for efficiency given in the video.

$e =$ __________ $e =$ ______

I. The transitive property says that you can equate the right sides of the two equations you wrote in your Part H answer. But, beyond that trivial result, WHY IS IT that those two “right sides” really are exactly equal to each other? HINT: It has to do with units.

Uβ, HW3, P5

Reference Videos: (1) "Conservative and Non-Conservative Forces"
(2) "Conservative and Non-Conservative Forces, Part II"
YouTube, lasseviren1, WORK, ENERGY, AND POWER playlist

A. With **conservative forces**, it is ___________ to get energy back out of the system; with **non-conservative forces**, it is ________________. With BOTH types of forces, energy ________________ ________.

B. When displacing an object from one specific point to a different specific point, the work done by a conservative force is…

The figures show a 4 kg mass being moved. In Figure I, which deals with gravity, our view is from the SIDE; the mass moves up, down, and horizontally. In Figure II, which deals with friction, the mass moves ONLY in the horizontal plane; we are LOOKING DOWN on the action. For convenience, the coordinate systems match. In all cases, the mass will start at (0, 0) and end at (3, 4). All coordinates are in meters.

C. If $g = 10\ \text{m/s}^2$, what is the force of gravity on this mass?

In Figure I, the mass moves from (0, 0) to (3, 4) via two different paths.

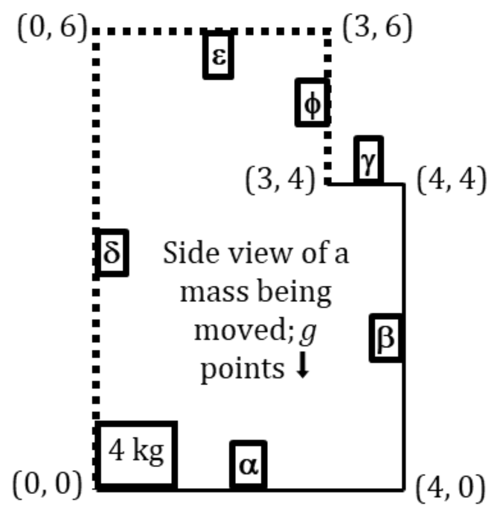


Figure I

D. Find the work done by gravity as the mass traverses segment:

i. α ii. β iii. γ

E. Now find the work done by gravity as the mass traverses segment:

i. δ ii. ε iii. ϕ

F. How do the sums of your Parts D and E answers compare?

Turn now to Figure 2. Assume the friction force is always 4 N.

G. Find the work done by friction as the mass traverses segment:

i. α ii. β iii. γ

H. Now find the work done by friction as the mass traverses segment:

i. δ ii. ε iii. ϕ

I. How do the sums of your Parts G and H answers compare?

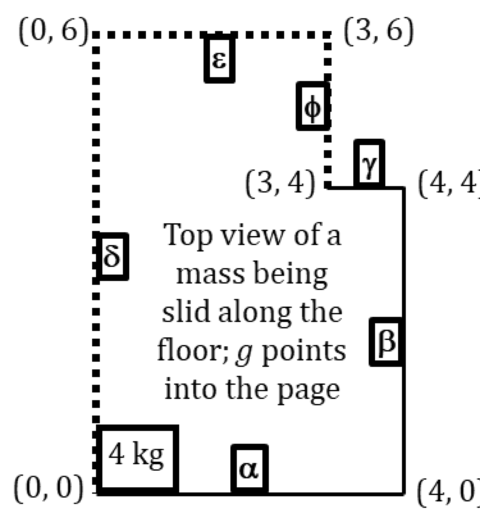


Figure II

J. Based on your answers to Parts B, F, and I, label these forces as 'conservative' or 'non-conservative'.

i. gravity ii. friction

K. In addition, the work done by any conservative force over a **closed path**…

L. Look again at both figures. If the mass proceeds CCW around the entire loop, starting at (0, 0) and ending at (0, 0), verify by calculation that your answers to Parts J and K are in agreement with regard to each type of force:

i. gravity ii. friction