APPC, Mechanics: Unit α HW 2

Name: ______________________________
Hr: ____ Due at beg of hr on: ____________

Uα, HW2, P1
Reference Videos: (1) "Derivatives and Antiderivatives, Part 2"
(2) "Derivatives and Antiderivatives, Part 3"
YouTube, lasseviren1

A. "Velocity is the time-derivative of the position function."
"Acceleration is the time-derivative of the velocity function."

velocity: $\vec{v}(t) =$

acceleration: $\vec{a}(t) =$

At right, write these two statements out, in mathematical terms.
HINT: You basically want your *rightmost* answers from Part H of HW 1, P1.

B. Determine the derivative of each function below. To do this – as the video showed – you need to differentiate EACH TERM in a given function. Begin each of your answers with what you are finding when you differentiate, i.e., each answer will start with either… $v(t) = \frac{dx}{dt} =$ OR $a(t) = \frac{dv}{dt} =$

i. $x(t) = 7t^2 + 5t - 3$

ii. $v(t) = -2t^3 - 3t + 1$

iii. $x(t) = -t^{-2} + t - 9$

iv. $v(t) = -2t^3 - 3t + 1$

v. $v(t) = 5t^4 + 3t^3 + 8t^2 - 4t + 7$

vi. $x(t) = 1.5t^4 - 3.5t^2 + 3.7t - 2.8$

vii. $v(t) = \frac{2}{t^3} - \frac{1}{t^2} - \frac{2}{t} - 6.8t$

viii. $x(t) = 3t^3 + 8t^2 - 13t - 28$

C. Notation such as $x(t) = \int v\,dt$ means either… "the __________________ of velocity with respect to time" OR "the ______________ ______________ of velocity with respect to time".

D. At right, list the two steps the narrator gives for integrating polynomial functions.

i.

ii.

E. Identify the name of each of the three parts of the integral at right. $\int \; 4t^2 \; dt$

F. Whenever you integrate, it is VERY important that you add WHAT additional portion (which you never have to do when you differentiate)?

G. An antiderivative of a function is related to WHAT FEATURE of a graph of that function?

Uα, HW2, P2

Reference Videos: (1) "Derivatives and Antiderivatives, Part 3"
(2) "Deriving Kinematics Equations Using Calculus"
YouTube, lasseviren1

A. "Position is the time-integral of the velocity function."
"Velocity is the time-integral of the acceleration function."
At right, spell these statements out, in mathematical terms.
HINT: Don't forget the *dt* on the right side of each equation.

position: $x(t) =$

velocity: $v(t) =$

B. Determine the antiderivative, i.e., the integral, of each function below. To do this, you need to integrate EACH TERM in the given function. Be sure to add the constant, i.e., + C , to each of your answers.

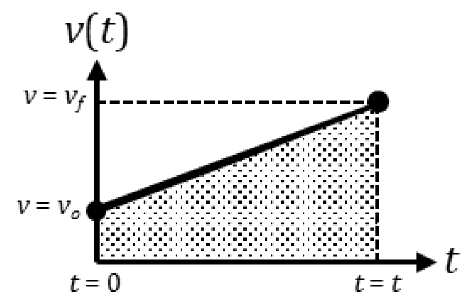
i. $\int 3t^2 dt$

iii. $\int (10t^4 + 2t^2 - \frac{3}{4}) dt$

ii. $\int (2t^5 - 7t^2) dt$

iv. $\int (4t^2 + 5t - 2) dt$

In Part B, you found <u>indefinite integrals</u>, which are integrals that don't have <u>bounds</u>, or <u>boundaries</u>, or <u>limits</u>, i.e., they have NO "bookend" points. Indefinite integrals always have the constant *C* at the end. <u>Definite integrals</u>, on the other hand, DO have endpoints. The *lower limit* is written at the bottom of the integral sign; the *upper limit*, at the top. A definite integral is always evaluated using these limits. When doing so, since the integral's lower limit value is subtracted from its upper limit value, the constant *C* cancels and so can be ignored. For example, in something such as $(U + C) - (L + C) = U - L$, notice that only the values of *U* and *L* are relevant; *C* falls away. This is what happens in a definite integral. At right is a graph to refer to as you complete this assignment.



C. Write the "calculus" or "differential" definition of acceleration.
This was one of your answers to HW2, P1, Part A.

D. Modify your Part C answer by moving the *dt* to the left side.

E. Now, as long as we do the exact same thing to both sides of an equation, the equation is still true. So, we will now show that we intend to integrate both sides of your Part D answer. We do this by (1) putting an integral sign at the leftmost part of each side of the equation, and (2) writing in the lower and upper limits below and above the integral sign, as described in the reading above and as shown in the second video. HINT: The limits (there are a total of four; two on each side of the equation) ARE the *x*- and *y*-coordinates that are shown in the graph above.

F. As shown in the second video, integrate both sides of your Part E answer: the left side with respect to *t*, and the right side with respect to *v*. On the right side of each integrated term, draw a right-end square bracket and write the same limits that you wrote in your Part E answer.

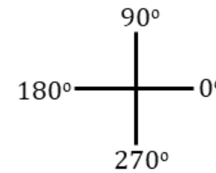
G. Now, evaluate both sides of the equation by plugging the limits into the expressions and subtracting the lower limit results from the upper limit results. Write out this work here.

H. Rearrange your Part G answer to yield Kinematics Equation I, as shown in HW1, P3.

Uα, HW2, P3
Reference Video: "A Few Thoughts on Vectors"
YouTube, lasseviren1

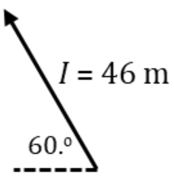
NOTE: There is no need to show work on this assignment; I trust that you understand the calculations. ☺

A. As shown in the video, write each of the vectors below in TWO different ways, namely:

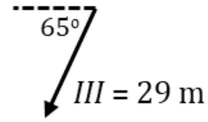


$\vec{V} = (magnitude, + \, degrees \; from \; 0^o)$ and $\vec{V} =$ % unit $\hat{\imath}$ $\pm$ # unit $\hat{\jmath}$

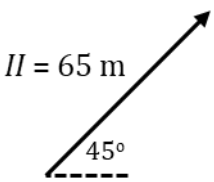
i.



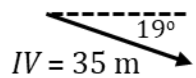
iii.



ii.



iv.



B. Add the $\hat{\imath}$ and $\hat{\jmath}$ vectors from your Part A answers to obtain the $\hat{\imath}$-and-$\hat{\jmath}$ form of the resultant vector $\vec{R} = \vec{I} + \vec{II} + \vec{III} + \vec{IV}$.

C. Use the Pythagorean theorem and the inverse tangent function to express your Part B answer in the form $\vec{R} = (magnitude, + \, degrees \; from \; 0^o)$.

D. A vector multiplied by (or divided by) a ________________ always yields another vector. Furthermore, these two vectors will then always have the exact same __________________.

E. Draw vector arrows over each vector quantity in the following equations. Leave scalar quantities as is.

i. $a_{avg} = \frac{\Delta v}{\Delta t}$

iii. $F_{net} = m \, a$

v. $v_f = v_o + at$

vii. $\Delta x = v_o t + \frac{1}{2} a t^2$

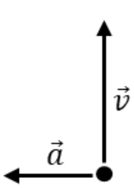
ii. $v_{avg} = \frac{\Delta x}{\Delta t}$

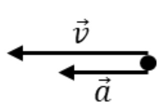
iv. $p = m \, v$

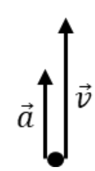
vi. $\Delta x = \frac{1}{2}(v_f + v_o)t$

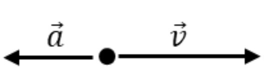
viii. $v_f^2 = v_o^2 + 2a\Delta x$

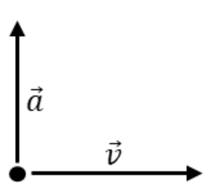
F. Write the correct answer for each of the seven cases below.
Your choices are: **speeding up** OR **slowing down** OR **turning at constant speed**

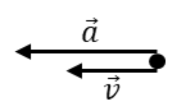










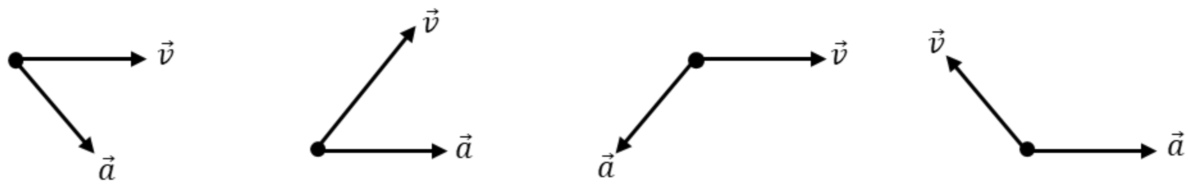


Uα, HW2, P4
Reference Video: "A Few Thoughts on Vectors (Part II)"
YouTube, lasseviren1

A. For each pair of velocity and acceleration vectors below:

i. Circle either "speeding up and turning" OR "slowing down and turning", and…

ii. Draw and label the centripetal ($\vec{a}_c$) and tangential ($\vec{a}_t$) acceleration vectors.



speeding up and turning	speeding up and turning	speeding up and turning	speeding up and turning
slowing down and turning	slowing down and turning	slowing down and turning	slowing down and turning

B. For each velocity vector shown below:

i. Draw and label an acceleration vector ($\vec{a}$) that satisfies the description below each vector, and…

ii. Draw and label the corresponding centripetal ($\vec{a}_c$) and tangential ($\vec{a}_t$) acceleration vectors.

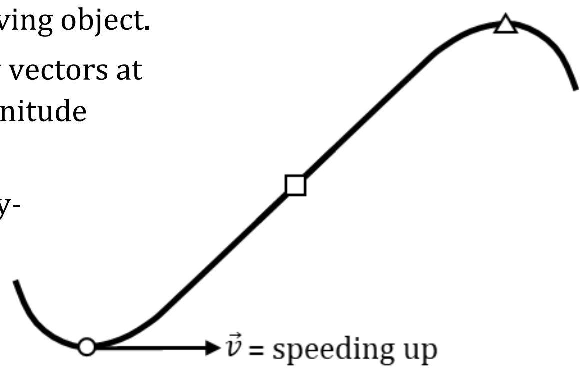
slowing down and turning	slowing down and turning	speeding up and turning	speeding up and turning

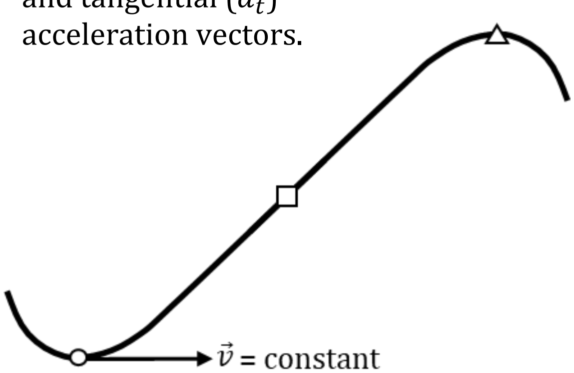
C. The three curves shown here represent the path of a moving object.

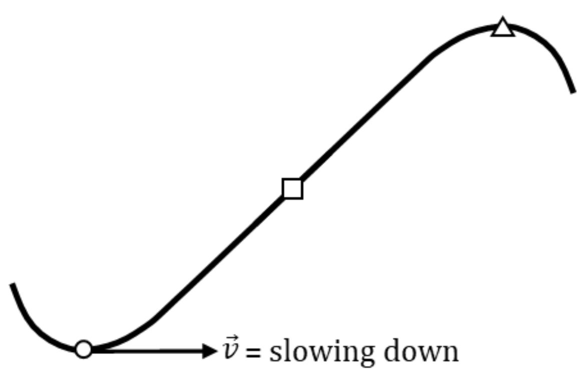
i. The velocity vector at point ○ is shown. Draw velocity vectors at all points □ and △, being sure to show A correct magnitude and THE correct direction at each location.

ii. At all locations ○, □, and △, draw in an appropriately-directed acceleration vector $\vec{a}$ IF the acceleration at that point is NONZERO. If an acceleration at some point IS zero, write "$\vec{a}$ = zero".

iii. Draw and label any corresponding centripetal ($\vec{a}_c$) and tangential ($\vec{a}_t$) acceleration vectors.







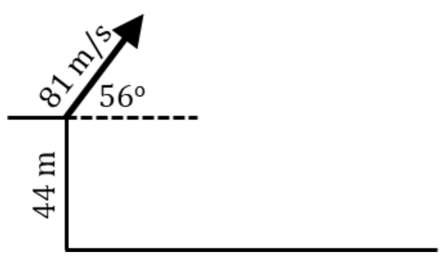
Uα, HW2, P5
Reference Video: "Projectile Motion"
YouTube, lasseviren1, KINEMATICS playlist

For each problem below, your goal is to obtain the projectile's **range**, so BOX that answer in. You will also need to find all other quantities that are asked for, but NOT necessarily in the order they are listed. The initial velocity and other constraints are given. Below each "Δy =' part, you need to show your calculation for time t, either by using $\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$ (i.e., solving a quadratic) OR – my personal preference – using $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ to first find v_{yf}, and then using $v_{yf} = v_{yi} + a_y t$ to obtain t. Assume that air resistance is negligible and that the acceleration due to gravity is 9.8 m/s². Also, employ proper sig figs in your 'range' answers; the other answers can (and should) have additional sig figs.

A.

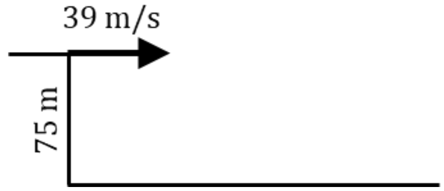
x	***y***
a_x =	a_y =
v_{xi} =	v_{yi} =
Δx =	Δy =

B.



x	***y***
a_x =	a_y =
v_{xi} =	v_{yi} =
Δx =	Δy =

C.



x	***y***
a_x =	a_y =
v_{xi} =	v_{yi} =
Δx =	Δy =