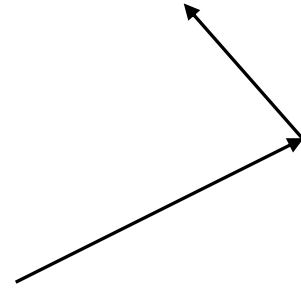


# Vectors and 2-D Motion

Name: \_\_\_\_\_

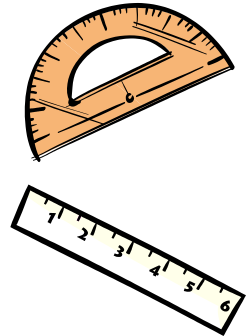
Video  
2d03  
(5:35)

resultant: the single vector that has the same effect as several vectors added together



## Vector Addition: Graphical Method

1. Choose a scale.
2. Use ruler and protractor to draw first vector to scale at proper angle.
3. Draw subsequent vectors tip-to-tail.
4. Draw resultant by connecting tail of first vector to tip of last.



EX. Add.  $d_1 = 420$  m horiz.  
 $d_2 = 280$  m @  $30^\circ$  above horiz. scale:



Video  
2d06  
(6:40)

Add.  $v_1 = 53$  km/h @  $25^\circ$  N of W  
 $v_2 = 32$  km/h @  $20^\circ$  S of W  
 $v_3 = 45$  km/h @  $25^\circ$  E of S

scale:



EX. Add.  $d_1 = 1160 \text{ km @ } 20.^\circ \text{ N of E}$   
 $d_2 = 470 \text{ km @ } 60.^\circ \text{ S of E}$   
 $d_3 = 280 \text{ km @ } 70.^\circ \text{ N of W}$

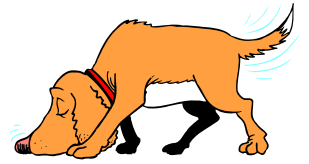
scale:



Video  
2d09  
(3:18)

Add.  $d_1 = 65 \text{ m @ } 15^\circ \text{ N of E}$   
 $d_2 = 48 \text{ m @ } 63^\circ \text{ N of E}$

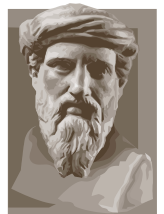
scale:



Video  
2d12  
(5:47)

### Vector Addition: Algebraic Method

If vectors are  $\perp$ , use Pythagorean theorem and trigonometry (i.e.,  $\tan^{-1}$ ).



EX. Add.  $d_1 = 75 \text{ km S}$   $d_2 = 45 \text{ km W}$



EX. Add.  $v_1 = 380 \text{ km/h N}$   $v_2 = 145 \text{ km/h E}$

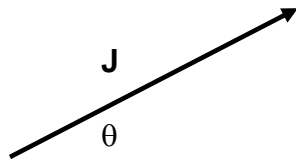


Video  
2d15  
(7:07)

### How to Add Vectors Algebraically if the Vectors are NOT $\perp$

vector resolution: taking a vector and breaking it down (“resolving” it) into two  $\perp$  component vectors

Use  $\theta$  along with sin and cos functions, as appropriate.



EX. Resolve into components.



$F = 68 \text{ N @ } 32^\circ \text{ above horiz.}$



$v = 14.7 \text{ m/s @ } 15.0^\circ \text{ below horiz.}$



$a = 3.2 \text{ m/s}^2 \text{ @ } 38^\circ \text{ S of W}$

Video  
2d18  
(7:21)

To add non-perpendicular vectors algebraically...

1. Resolve all vectors into “x” and “y” components.
2. Add “x” components, taking into account direction of each.
3. Add “y” components in the same way.
4. Vectors resulting from Steps 2 and 3 are  $\perp$ . Add them as you would any two  $\perp$  vectors, using Pythagoras and  $\tan^{-1}$ .



EX. Add algebraically.

$d_1 = 65 \text{ m @ } 15^\circ \text{ N of E}$

$d_2 = 48 \text{ m @ } 63^\circ \text{ N of E}$



$$d_{1EW} =$$

$$d_{2EW} =$$

$$d_{1NS} =$$

$$d_{2NS} =$$

$$\text{add comp.'s } \begin{cases} d_{rEW} = \\ d_{rNS} = \end{cases}$$

$$d_r =$$

$$\theta =$$



Add algebraically.

$d_1 = 425 \text{ km @ } 12^\circ \text{ E of N}$

$d_2 = 182 \text{ km @ } 28^\circ \text{ E of S}$



$$d_{1EW} =$$

$$d_{2EW} =$$

$$d_{1NS} =$$

$$d_{2NS} =$$

$$\text{add comp.'s } \begin{cases} d_{rEW} = \\ d_{rNS} = \end{cases}$$

$$d_r =$$

$$\theta =$$

Video  
2d24  
(7:01)

## Projectile Motion

projectile: an airborne object acted upon "only" by gravity

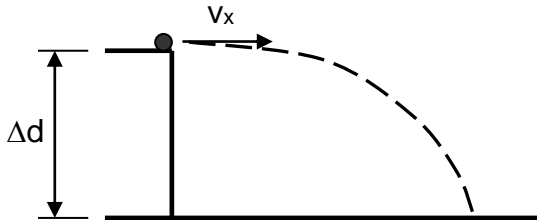
-- other forces are NOT significant

examples:

non-examples:

### Horizontally-Launched Projectiles

Theory: A projectile is launched horizontally w/velocity  $v_x$  from height  $\Delta d$ .



--  $v_x$  is...

--  $a_x$  is...

--  $v_y$  starts at...

#### Kinematics Equations:

$$\mathbf{A} \quad \vec{a} = \frac{\vec{\Delta v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\mathbf{B} \quad \vec{v}_f^2 = \vec{v}_i^2 + 2 \vec{a} \Delta d$$

$$\mathbf{C} \quad \vec{v}_{\text{avg}} = \frac{\vec{\Delta d}}{\Delta t} = \frac{1}{2} (\vec{v}_f + \vec{v}_i)$$

$$\mathbf{D} \quad \vec{\Delta d} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} (\Delta t)^2$$

Video  
2d27  
(8:09)

A rifle bullet is fired horizontally w/initial speed 850 m/s from height 1.73 m. Find...

a. ...time bullet is in air

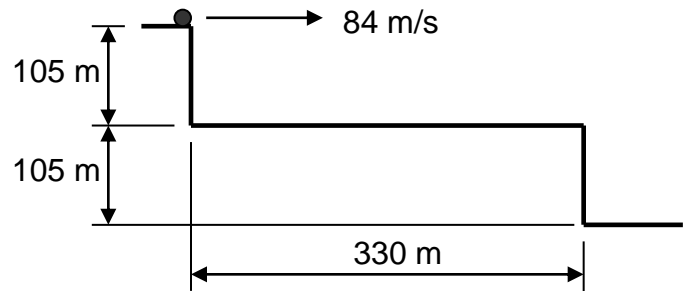


b. ...bullet's range (max. horizontal displacement)

c. ...bullet's height above ground at  $t = 0.38$  s.

Video  
2d30  
(5:54)

For how long will projectile remain airborne?



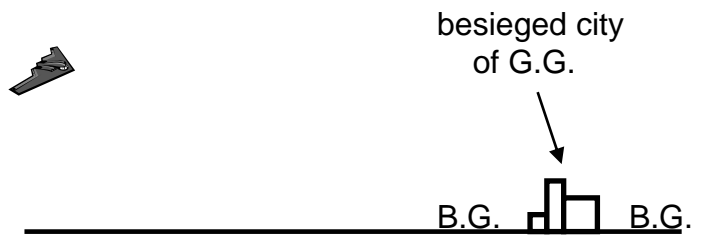
Video  
2d33  
(6:24)

Projectile is launched horizontally from height 62 m. Range is measured to be 238 m. Find launch speed.



Video  
2d36  
(7:01)

Military plane flies horizontally at altitude 8000. m at speed 100. m/s. As measured along ground, how far from target must plane be when it releases target's provisions?



How fast are provisions moving when they land?

Video  
2d39  
(5:07)

### Review Problem from 1-D Motion

An object is launched w/initial vel.  $37.0 \text{ m/s}$  ↑.

a. Find time object is in air.

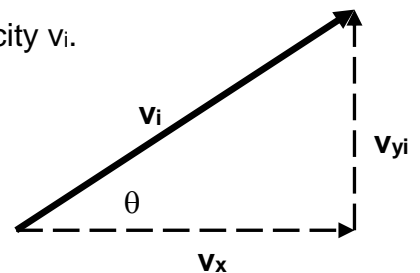
b. Find max. height object attains.



### Projectiles Launched at an Angle

Theory: A projectile is launched at initial angle  $\theta$  with initial velocity  $v_i$ .

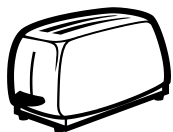
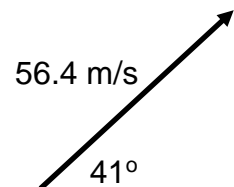
- $v_x$  is...
- $a_x$  is...
- $v_y$  starts at...



Video  
2d42  
(6:52)

Projectile is launched w/initial velocity  $56.4 \text{ m/s}$  at  $41^\circ$  above horizontal.

a. Find time object is in air.



b. Find max. height object attains.

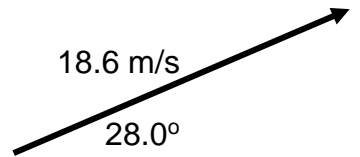
c. Find projectile's range.

An angled projectile...

Video  
2d45  
(4:40)

Projectile is launched w/initial velocity 18.6 m/s at  $28.0^\circ$  above horizontal.

a. Find time object is in air.



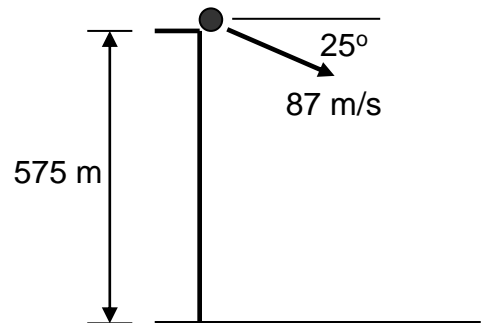
b. Find max. height object attains.

c. Find projectile's range.

Video  
2d48  
(4:20)

Projectile is launched from height 575 m w/initial velocity 87 m/s @  $25^\circ$  below horizontal.

a. Find time object is in air.



b. Find projectile's range.



Video  
2d51  
(7:45)

## At Any Time $\Delta t$ After Launch, WHERE is a Projectile?

Use THIS equation...

...twice.

The x-position will have CHANGED by the amount...

The y-position will have CHANGED by the amount...

These changes, along with the initial x- and y-positions...

EX. An object is launched from the ground w/init. velocity 35.58 m/s @ 55.80° above the horizontal. Find its location relative to its starting point 2.43 s after launch.



EX. Proj. is launched horizontally at 36 m/s from height 92 m. Find its location 3.24 s after launch.



Video  
2d54  
(8:01)

## At Any Time $\Delta t$ After Launch, What's a Projectile's VELOCITY?

Use THIS equation...

...once

$\vec{v}_x$ : Horiz. ( $v_x$ ) is...

To find resultant  $v_r$  at any time  $\Delta t$ :

$\vec{v}_y$ : Vert. ( $v_y$ ) is...

EX. An object is launched from the ground w/init. velocity 35.58 m/s @  $55.80^\circ$  above the horizontal. Find its velocity 2.43 s after launch.



EX. Projectile is launched horizontally at 36 m/s from height 92 m. Find its velocity 3.24 s after launch.



Video  
2d57  
(8:18)

A cannonball is launched off the edge of a cliff at an init. speed of 49.5 m/s at  $25.6^\circ$  above horiz. The ball is in the air for 9.75 s. Find height of cliff, the range, and the impact speed of the cannonball.