Name: _____
Hour: ____ Date: _____

## Physics: *Linear Regression (Linear Least Squares Fit)*

1. A cart is released with some nonzero initial velocity. Its velocity at each time t after release is given below.

| X [t (s)] | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 |
|---|---|---|---|---|---|---|
| Y [$v_f$ (m/s)] | 0.843 | 1.051 | 1.298 | 1.462 | 1.734 | 1.925 |

a. Find the equation of the linear regression line that describes this data.

b. The slope of a velocity-time curve is the object's acceleration. What is the cart's acceleration?

c. Estimate the initial velocity of the cart (i.e., the velocity at t = 0.00 s).

d. Estimate the time at which the velocity of the cart is 5.634 m/s.

2. A ball is released from rest. Its position at each time t after release is given below.

| [t (s)] | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 |
|---|---|---|---|---|---|---|
| X [$t^2$ ($s^2$)] | | | | | | |
| Y [y (m)] | 0.303 | 1.234 | 2.768 | 4.892 | 7.651 | 11.105 |

a. Find $t^2$ for each data point and fill in the table.

b. Find the equation of the linear regression line that describes this data.

c. The acceleration of the ball is equal to twice the slope of the line. Find the acceleration.

d. Based on the acceleration you found in part c, describe the motion of the ball.

e. Based on your linear regression equation, estimate the position of the ball at t = 3.20 s.

3. The volume of a confined sample of gas is measured at various temperatures, as shown below.

| X [T ($^o$C)] | –40 | –10 | 0 | 20 | 30 | 40 |
|---|---|---|---|---|---|---|
| Y [V ($cm^3$)] | 22.01 | 24.84 | 25.55 | 27.73 | 28.51 | 29.68 |

a. Find the equation of the linear regression line that describes this data.

b. Estimate the volume of the sample when the temperature is 125$^o$C.

c. Estimate the temperature at which the volume of the sample is zero.

4. Two measurements, X and Y, are made on geometric figures of identical shape but varying size.

| X (m) | 0.132 | 0.288 | 0.374 | 0.621 | 0.873 | 1.056 |
|---|---|---|---|---|---|---|
| Y (m) | 0.412 | 0.908 | 1.169 | 1.947 | 2.749 | 3.320 |

a. Find the equation of the linear regression line that describes this data.

b. What type of geometric figure is being measured?

c. What dimension of the figure does X represent?

…Y represent?

5. A ball attached to a string is whirled in a horizontal circle. The radius of the motion and the speed of the ball are given below for several trials.

| X [R (m)] | 0.20 | 0.40 | 0.60 | 0.80 | 1.00 | 1.20 |
|---|---|---|---|---|---|---|
| [v (m/s)] | 0.814 | 1.141 | 1.407 | 1.588 | 1.824 | 2.012 |
| Y [$v^2$ ($m^2/s^2$)] | | | | | | |

a. Find $v^2$ for each data point and fill in the table.

b. Find the equation of the linear regression line that describes this data.

c. The acceleration of the ball is the slope of the line. Find the acceleration.

d. Estimate the speed of the ball when the circle's radius is 0.13 m.

e. Estimate the circle's radius when the speed of the ball is 2.756 m/s.

6. The amount of energy contained within a given mass is given by the data below

| X [m (kg)] | 0.100 | 0.200 | 0.400 | 0.600 | 0.800 | 1.000 |
|---|---|---|---|---|---|---|
| Y [E (J)] | $8.96 \times 10^{15}$ | $18.0 \times 10^{15}$ | $35.7 \times 10^{15}$ | $54.1 \times 10^{15}$ | $72.2 \times 10^{15}$ | $90.0 \times 10^{15}$ |

a. Find the equation of the linear regression line that describes this data.

b. The slope has units of $m^2/s^2$. Calculate the square root of the slope and tell what it represents.

c. If a football field were covered in 10 feet of solid ice, it would take $4 \times 10^{13}$ J of energy to boil away all of that ice. Estimate the mass of matter (in grams) that contains this much energy.

## Linear Regression worksheet answers:

1a. $v_f = 0.8712t + 0.6232$
1b. $0.8712\ m/s^2$
1c. $0.6232\ m/s$
1d. $5.75\ s$
2a. $0.0625\ s^2$, $0.25\ s^2$, $0.5625\ s^2$, $1.00\ s^2$, $1.5625\ s^2$, $2.25\ s^2$
2b. $y = 4.926t^2 – 0.011$
2c. $9.852\ m/s^2$
2d. free fall
2e. 50.43 m
3a. $V = 0.0953T + 25.75$
3b. $37.66\ cm^3$
3c. $–270.2^oC$
4a. $Y = 3.148X – 0.004$
4b. circle
4c. diameter; circumference
5a. $0.663\ m^2/s^2$, $1.302\ m^2/s^2$, $1.980\ m^2/s^2$, $2.522\ m^2/s^2$, $3.327\ m^2/s^2$, $4.048\ m^2/s^2$
5b. $v^2 = 3.363R – 0.0472$
5c. $3.363\ m/s^2$
5d. $0.624\ m/s$
5e. 2.27 m
6a. $E = 9.019 \times 10^{16}\ m – 1.058 \times 10^{14}$
6b. $3.00 \times 10^8\ m/s$; speed of light
6c. $m = 1.6\ g$


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