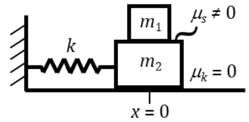
APPC, Mechanics: Unit δ HW 3

Name: ______________________________

Hr: ___ Due at beg of hr on: __________

Uδ, HW3, P1

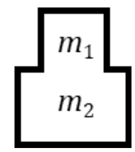
Reference Videos: (1) "Harmonic Oscillator with Two Objects"
(2) "Springs in Series and Parallel"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



In the figure above-right, we wish to determine the maximum displacement x such that m_1 does NOT slide, relative to m_2. There IS friction between the masses, but none between m_2 and the surface. For maximum displacement, it follows that we must max out the static friction force between m_1 and m_2 .

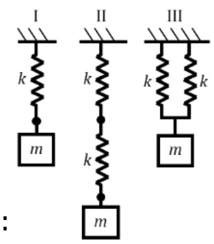
A. Let us displace the system this maximum amount $+x$, i.e., to the right. (Any larger displacement would give an acceleration too large, and m_1 would slide.) Using Hooke's law, write an expression for the elastic force F_{elas}.

B. Considering ($m_1 + m_2$) as a combined mass, use the figure at right to draw an FBD of this combined mass. Then, using your FBD and your Part A answer, write a Newton's 2nd law equation and solve it for the acceleration of the two-mass system.



C.

Now, consider only m_1. Draw an FBD using the figure at left. Again, write a Newton's 2nd law equation and solve it for the acceleration of m_1.



D. If the two masses are to stay together, any part of the system must have the same acceleration, i.e., your Parts B and C answers must be equal to each other. Do that, and solve for x.

E. Based on the figure, CIRCLE the answers to these questions about springs. Which system:

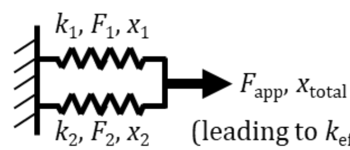
i. ...is the "tightest"? I II III

ii. ...is the "flopsy-est"? I II III

iii. ...will take the most time for one oscillation? I II III

iv. ...will take the least time for one oscillation? I II III

Now, we turn to finding the effective spring constant k_{eff} for multiple-spring systems. Consider the figure, where a force F_{app} is applied to the system shown, resulting in forces and displacements in both springs.



(leading to k_{eff} for the two-spring system)

F. The springs shown are in: (CIRCLE) SERIES PARALLEL

G. When F_{app} is applied, which TWO choices will be true? (CIRCLE <u>TWO</u>)

I. $x_1 + x_2 = x_{total}$ II. $x_1 > x_2$ III. $x_1 < x_2$ IV. $x_1 = x_2 = x_{total}$, i.e., x for all parts of the system

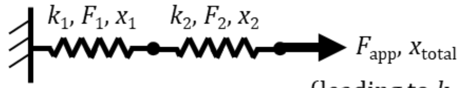
V. $F_1 + F_2 = F_{app}$ VI. $F_1 > F_2$ VII. $F_1 < F_2$ VIII. $F_1 = F_2 = F_{app}$, i.e., F for all parts of the system

H. One of your Part G answers should have a (+) symbol. Use Hooke's law to transform that answer's equation FROM the *one type of variable that it already has* INTO *the other two types of variables that are in Hooke's law*.

I. Your other Part G answer should have all (=) signs, i.e., NOT three discrete variables, only one. Substitute this ONE variable into your Part H answer, then simplify/cancel as needed. Show your work.

J. If you had three springs configured as above (with spring constants k_1 , k_2 , and k_3), what would the equation be to find k_{eff} for the system?

Uδ, HW3, P2
Reference Videos (1) “Springs in Series and Parallel”
(2) “VID00129 (Period of a Simple Pendulum)”
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



(leading to k_{eff} for the two-spring system)

The figure shows a force F_{app} applied to the system, resulting in forces and displacements in both springs.

A. The springs shown are in: (CIRCLE) SERIES PARALLEL

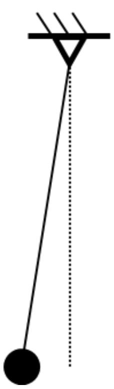
B. When F_{app} is applied, which TWO choices will be true? (CIRCLE TWO)

I. $x_1 + x_2 = x_{total}$ II. $x_1 > x_2$ III. $x_1 < x_2$ IV. $x_1 = x_2 = x_{total}$, i.e., x for all parts of the system

V. $F_1 + F_2 = F_{app}$ VI. $F_1 > F_2$ VII. $F_1 < F_2$ VIII. $F_1 = F_2 = F_{app}$, i.e., F for all parts of the system

C. One of your Part B answers should have a (+) symbol. Use Hooke’s law to transform that answer’s equation FROM the *one type of variable that it already has* INTO *the other two types of variables that are in Hooke’s law*.

D. Your other Part B answer should have all (=) signs, i.e., NOT three discrete variables, only one. Substitute this ONE variable into your Part C answer, then simplify/cancel as needed. Show your work.



E. If you had three springs configured as above (with spring constants k_1, k_2, and k_3), what would the equation be to find k_{eff} for the system?

F. A ____________ pendulum is one in which _____ of the mass is considered to be a point mass and is located at the end of the string, i.e., the weight of the ___________ is negligible. In addition, the __________ of displacement of the pendulum must be small, i.e., no greater than 15°.

G. In the figure, label the quantities m, L, x, and θ.

H. Below-right, draw an FBD of the mass when it is at the position shown in the figure.

I. In your FBD, break the mg vector into components // and ⊥ to the bob’s velocity. Label them with what each component would be numerically equal to, taking into account the angle θ. Also, draw axes for reference. Label the axes // and ⊥ .

J. Use Newton’s 2nd law in the // direction. Solve for the acceleration // to the velocity, $a_{//}$.

K. What type of acceleration is your Part J answer? (CIRCLE) CENTRIPETAL TANGENTIAL

L. Based on the definition of the *sin* function, modify your Part J answer to eliminate the θ.

M. Rewrite your Part L answer by placing a (−) sign in front of the right side.

N. If θ is small (< 15°), the directions of the displacement x and your acceleration of Part J are very nearly parallel. With this in mind, why was it okay for us to insert the ‘extra’ (−) sign in Part M? HINT: Think about where $x = 0$.

O. In HW2, P3, we found that, for SHM, $a(t) = -\omega^2\, x(t)$; that is, at any point, $a = -\omega^2\, x$. Substitute this expression into your Part M answer and solve for ω^2.

P. Combine your Part O answer with an equation you wrote in Part A of HW2, P4 to derive a (hopefully) recognizable equation for the period T of a simple pendulum.

Q. Finally, why must it be true that $T \neq mg\cos\theta$ in your FBD and that, in fact, T must be greater than $mg\cos\theta$? HINT: Your Part K answer should somehow point you in the right direction.

Uδ, HW3, P3
Reference Video: "Period of a Physical Pendulum"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

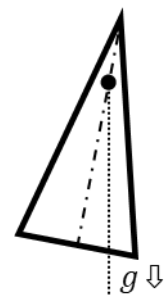
A. A ____________________ pendulum is one in which NOT all of the ________ is assumed to be concentrated at a single point, but rather is distributed in some fashion relative to the pendulum's ________.

B. In HW2, P4, we found that, in SHM, $a = -\omega^2 x$. Write the rotational analog of this equation.

C. Write the Maclaurin series expansion (which, FYI, is a Taylor series centered at x = 0...Who cares?!?) for sin x. Write SIX terms on the equation's right side.

D. Simplify your Part C equation to essentially what it becomes when x is very tiny.

E. Here, we are dealing NOT with x, but with its rotational analog. Rewrite your Part D answer, accounting for this fact.



An object of mass m, uniform density, and rotational inertia I is to oscillate about an axis near the top of the object. The straight-downward direction is indicated by the dotted line, and the object's axis of symmetry is indicated by the dot-and-dash line.

F. Into the figure, draw and label a ⊙ approximately where the object's *com* is located.

G. The distance between an object's *com* and its rotational axis is usually assigned the variable d. Using dimension lines and a two-headed arrow, label this dimension in the figure. Also, use whatever means necessary (arrows or otherwise) to label the displacement angle θ in the figure.

H. Write the equation for the net torque that m's weight exerts, about the axis.

I. Taking into account your Part E answer, rewrite your Part H answer.

J. One translational form of Newton's 2nd law is $F_{net} = ma$. Write the rotational analog of this equation.

K. Substitute your Part I answer into your Part J answer.

L. Now, refer back to the figure, then CIRCLE your answers.

i. When θ is ↪ from the equilibrium position, the angular acceleration is directed: ↪ ↩

ii. When θ is ↩ from the equilibrium position, the angular acceleration is directed: ↪ ↩

M. Ever-so-slightly, modify your Part K answer, in order to account for your Part L responses.

N. Substitute your Part B answer into your Part M answer. Solve for angular frequency.

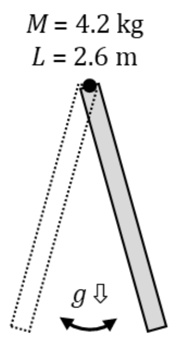
O. Combine your Part N answer with an equation you wrote in Part A of HW2, P4 to derive an equation for the period T of a physical pendulum.

P. To reiterate, your Part O equation holds ONLY when angular displacements have magnitudes that are very ________.

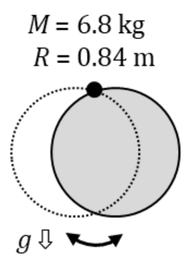
Uδ, HW3, P4
Reference Video: "Physical Pendulum (Part II)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

Determine the period of oscillation for each pendulum below. All objects are uniform masses. Show your work. BOX in your answers, and round them to THREE sig figs. Use 9.8 m/s² for g. In determining moments of inertia, it may be helpful to look back at your work from Uγ, HW2, P5 through HW3, P3.

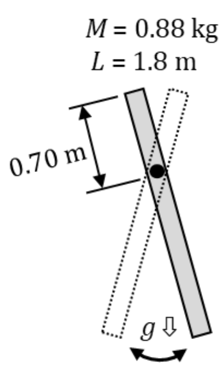
A. Solid rod, pinned at one end



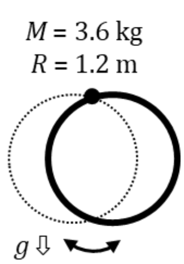
B. Solid disk, pinned at an edge



C. Solid rod, pinned at location shown

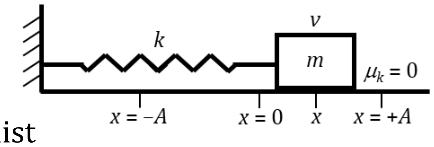


D. Thin ring, pinned at an edge



Uδ, HW3, P5

Reference Video: "Vertical and Horizontal Harmonic Oscillators"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

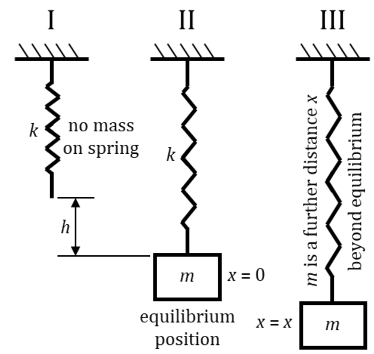


Many students are comfortable dealing with horizontal mass-spring systems, but get confused if the system is oriented vertically. We will attempt to overcome this potential hang-up here.

A. With reference to the figure above-right... Draw an FBD of m when it is at the location x. Employ Hooke's law, putting the associated expression into the FBD.

B. Write a NO-denominators Newton's 2nd law equation for the x-direction.

C. (EASY!) How does the magnitude of the net force on m compare to the magnitude of the elastic force in the spring?



Let's take the same system and turn it vertical, so k and m are the same as before. In State I, no mass hangs from the spring (which we assume is massless). In State II, m hangs from the spring at equilibrium, i.e., State II is how the system would look if m were simply hanging, at rest. You see that, in State II, m's weight has extended the spring a distance h. State III shows m an additional distance x below the equilibrium position. Basically, we have set m to oscillating up and down, and State III is just an instantaneous snapshot showing when m happens to be at the NON-maximum displacement x.

D. Draw an FBD of State II and write a NO-denominators Newton's 2nd law equation. Obviously, any time m is in State II (whether its $v = 0$ or not), m will NOT be accelerating.

E. Draw an FBD of State III and write an unsimplified, NO-denominators Newton's 2nd law equation. HINT: Your answer should have one (+), one (−), and one set of ( ).

F. Get rid of the ( ) in your Part E answer by employing the distributive property.

G. Substitute your Part D answer into your Part F answer and simplify.

H. How do your Parts B and G answers compare?

I. Your Part H answer says that, whether you have a horizontal- OR a vertical mass-spring system, the NET force on m when it is a displacement x from equilibrium always boils down to $F_{net} = ma =$ ____. However, the ELASTIC force in the spring for a horizontal system is equal to ____ (HINT: See your FBD in Part A) while, for a vertical system, the elastic force is equal to ________ (HINT: See your FBD in Part E). Furthermore, since *equilibrium* means something to the effect of 'the location where a system could remain motionless, indefinitely, without changing', then a horizontal system is in equilibrium at $x =$ ___, whereas a vertical system is in equilibrium, in our case above, at $x =$ ___. And because the elastic force is NOT the same for horizontal versus vertical, as well as because the equilibrium position is NOT the same, neither will the ______________ for the two scenarios be the same.