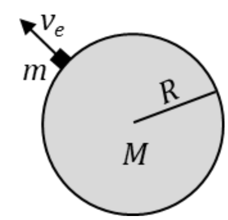
APPC, Mechanics: Unit δ HW 2

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

Uδ, HW2, P1
Reference Videos: (1) "Escape Velocity"
(2) "Binary Star Systems"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

A. **Escape velocity** is the ________________ launching velocity required for an object to ________ __________. (This assumes that no ___________ is added along the way.) To review, all kinetic energies have a sign that is ___, and all absolute gravitational potential energies have a sign that is ___. So, if we don't want to be ________ to Earth, then we need to have a total energy that is at least ______.

In the figure, a mass m is launched with escape velocity v_e from the surface of a celestial body of mass M and radius R. We want a general expression for v_e.



B. Using the variables v_e and R, write an equation depicting the total energy m will have at the instant of lift-off. This is the total energy it will have when it is a distance R from M's center, i.e., $E_{r=R}$ = ?

C. After m is launched at a speed of v_e, its speed thereafter will ________________ with time.

D. Explain your Part C answer. Make reference to one or more aspects of Newton's 2nd law.

E. For the conditions of escape velocity to be satisfied, m must reach a distance of $r = \infty$. Using the variables $v_{r=\infty}$ and $r_{r=\infty}$ (☺), write an equation that depicts the total energy m will have at $r = \infty$, i.e., $E_{r=\infty} =$?

F. Since we will be conserving energy, use that fact to combine your Parts B and E answers so that relevant terms of the situation are still visible. Do NOT yet simplify.

G. *Mentally* modify the right side of your Part F answer by carrying your Part C answer to its logical conclusion AND by inputting information from the Part E question. Having accounted for these modifications, write your modified Part F equation here.

H. Simplify your Part G answer to obtain an expression for v_e.

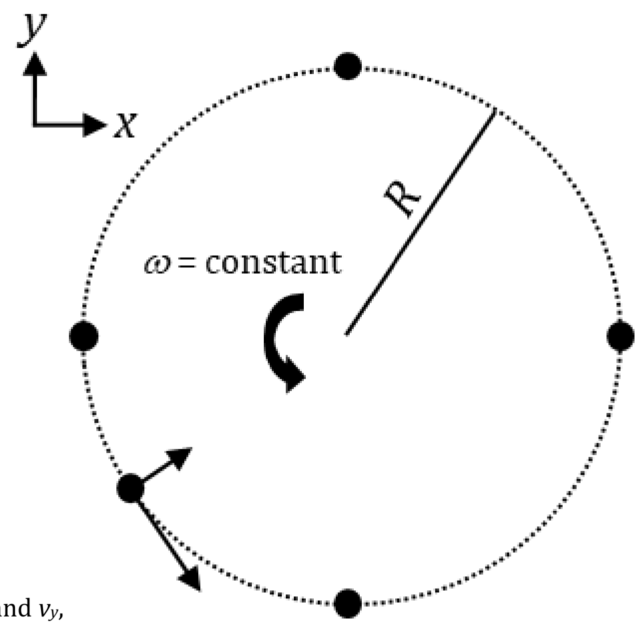
I. A **binary star system** consists of two stars that revolve about their common __________ ___ ________ (i.e., their _____). We believe that _______ stars in our universe exist in binary star systems.

J. Assuming $M > m$, depict (and label) the first sentence of your Part I answer using the figure below. Show the orbital paths (assume circular) for each star and label the radii of the orbits as r_m and r_M.

K. CIRCLE which star has the larger:

i. period of motion	m	M	it's a tie
ii. tangential velocity	m	M	it's a tie
iii. force of gravity on it	m	M	it's a tie
iv. acceleration	m	M	it's a tie

Uδ, HW2, P2
Reference Video: "Kinematics of Simple Harmonic Motion (Part I)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



NOTE: *The line of reasoning here and in subsequent assignments differs slightly from the videos. Nonetheless, you will benefit from having watched those in advance of trying the next few assignments.*

The figure shows an object in uniform circular motion (UCM).
PLEASE IMAGINE that the *xy*-axes shown are at the circle's center.

A. Make the following additions to the figure.

i. One point on the circle already has two vectors. Label these (correctly!) as $\vec{v}$ and $\vec{a}$.

ii. At the other four designated points, draw and label $\vec{v}$ and $\vec{a}$ vectors of the correct direction and length.

iii. On the pre-drawn $\vec{v}$ vector (see Part Ai), draw components v_x and v_y, based on the *xy*-axes shown. (Again, imagine that these are at the center).

iv. On the pre-drawn radius R, draw in and label its components as x and y.

v. Label the angle between R and x as θ.

B. Let's digress for a minute... Write the:

i. velocity bridge equation

ii. acceleration bridge equation

C. Write our often-used equation for centripetal acceleration.

D. Substitute the right side of your Part Bi answer into your Part C answer and then simplify to obtain a (for us) lesser-used equation for centripetal acceleration.

E. One last thing: Write what x is equal to, in terms of R and θ. (Look again at the figure.)

F. Apply the next steps to ONLY the figure's four 'compass points', i.e., NOT to the point with the pre-drawn $\vec{v}$ and $\vec{a}$ vectors.

i. Put the appropriate subscript, either $_x$ or $_y$, onto those points' $\vec{v}$ and $\vec{a}$ labels.

ii. Enclose each of the $\vec{v}$ and $\vec{a}$ labels in absolute value signs and then write " $= max$ " next to each one. Verify mentally that this claim is true for each of the compass-point $\vec{v}$ and $\vec{a}$ vectors.

iii. TWO of these four [$a_x = 0$ $a_y = 0$ $v_x = 0$ $v_y = 0$] apply to each compass point. Write the correct two of them near each point.

iv. ONE of these two [$|x| = max$ $x = 0$] applies to each compass point. Write the correct one near each point.

G. CIRCLE every quantity in the figure that refers to the x-direction.

Let's digress again... Since $velocity = \frac{displacement}{time}$, we recall that $\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t}$.
If we rename θ_f as just θ and θ_i as ϕ, we get... $\omega = \frac{\theta - \varphi}{t}$.

H. Solve this last expression for θ.

I. Substitute your Part H answer into your Part E answer.

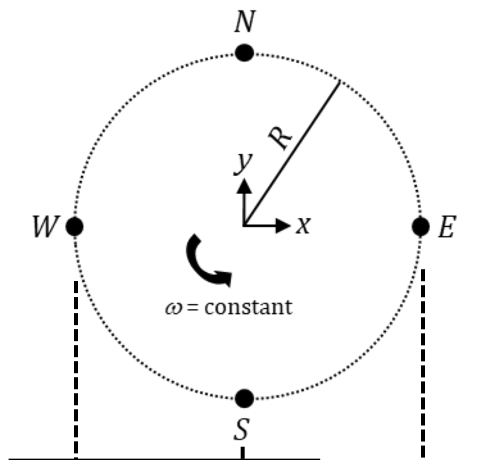
J. Your Part I answer is an equation for x-position vs. time. If you differentiate this once, you'll get an equation for the x-______________ vs. time; if you differentiate it once more, you'll get an equation for the x-__________________ vs. time.

K. Differentiate your Part I answer: i. ...once here... ii. ...and once more over here.

(Your answers will agree with the video, with the notable exceptions that you'll have R instead of A and $(\omega t + \phi)$ instead of (ωt).

Uδ, HW2, P3

Reference (1) “Kinematics of Simple Harmonic Motion (Part I)”
Videos: (2) “Kinematics of Simple Harmonic Motion (Part II)”
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



We continue from HW2, P2...

A. The maximum value that a *sin* or *cos* function can have is ____.

B. Given your Part A answer, your Parts I and K answers from P2 reveal that, for the circular motion depicted at the top of that assignment and partially reproduced in the figure at right...

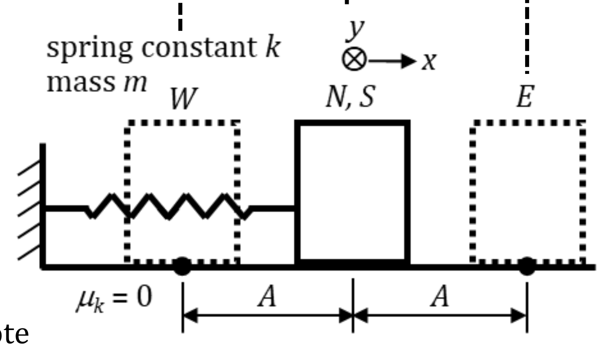
i. $|x_{max}| =$ ____ ...which occurs at the following TWO points: N S E W

ii. $|v_{x,max}| =$ ______ ...which occurs at the following TWO points: N S E W (CIRCLE)

iii. $|a_{x,max}| =$ _______ ...which occurs at the following TWO points: N S E W

C. State how what you wrote in the blanks in Parts Bii and Biii above compares to Parts Bi and D from P2.

D. State how what you circled in Part B compares to the MAX x-quantities you circled in Part G on P2.



The thinking and mathematical formulating we’ve done so far in P2 and P3 (specifically, in the x-direction) correspond EXACTLY to the kinematics of a mass m oscillating on a spring of constant k. The figure at right shows such a case, on a frictionless surface. It is as if the figure at the top of the page is a ‘bird’s eye’ view and the figure here is an elevation view. Note here how the same xy-axes are designated. Note also the ‘drop-down’ lines for points N, S, E, and W.

E. Write THREE of the following six choices onto EACH depiction of the mass in the figure.

$|x| = max$ $\quad x = 0 \quad$ $|v_x| = max$ $\quad v_x = 0 \quad$ $|a_x| = max$ $\quad a_x = 0$

F. State how your Part E answers compare to your Part B answers.

G. Look at both figures: What is the mathematical relationship between A and R?

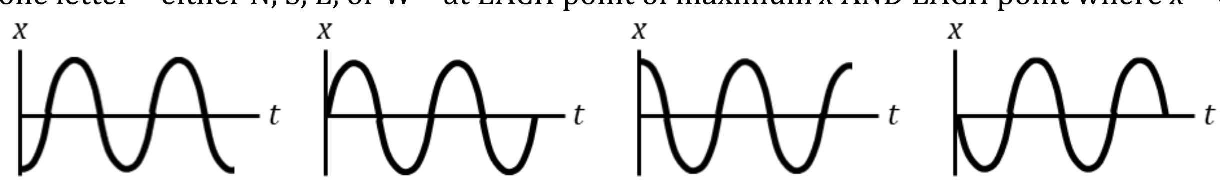
H. State how what you wrote in the blank in Part Bi compares to your Part G answer.

I. Write three equations by substituting your Part G answer into your Parts I and K answers from P2.

$x(t) =$ $\qquad$ $v(t) =$ $\qquad$ $a(t) =$

*** Your Part I answers are the kinematics equations for simple harmonic motion (SHM) of a mass on a spring. Hopefully, now, you see the correspondence between the SHM of a mass on a spring and UCM.*

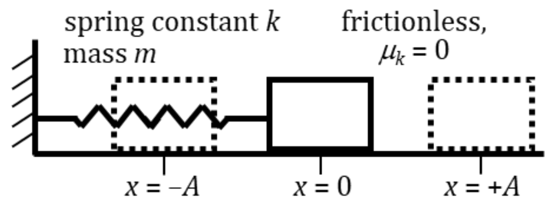
J. The ϕ term in your Part I answers has to do with WHERE m starts on its back-and-forth journey, i.e., when t = 0. (More on this later.) The graphs below depict position-vs.-time for m starting at various points. Your task here is to (1) figure out how the graphs correspond to the two figures above, and (2) write one letter – either N, S, E, or W – at EACH point of maximum x AND EACH point where x = 0.



Uδ, HW2, P4
Reference Videos: (1) "Kinematics of Simple Harmonic Motion (Part III)"
(2) "Dynamics of Simple Harmonic Motion"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

A. In UCM and SHM, the **period** T is essentially the number of ______________ for one __________, i.e., the _______ for one ___________ _______. (So, obviously, the best SI unit for period T is abbreviated ___.) The **frequency** f is the number of ___________ per ____________, and is often measured in the unit _________, which is abbreviated ____. A mathematical equation that relates period T to frequency f is ________. Specifically in UCM or SHM, the variable ω is called the _______________ ____________________ and has the unit of ______________ per ____________. In the sinusoidal curves that we met in the last assignment, one complete cycle of UCM or SHM is equivalent to ____ radians of angular displacement. Therefore, if we focus on the period of a single cycle, we see that ω = ______; if we focus on frequency, then ω = ________ .

The figure at right shows a mass that oscillates in SHM.



B. Below the figure, draw an FBD of the mass when it is located at some distance x between $x = 0$ and $x = +A$. All forces (one of them is F_{elas}) should originate on the *com* of the object.

C. Hooke's law is $F_{elas} = -k\,x$. The elastic force is one type of **restoring force**, which is any force whose magnitude is ______________ ______________________ to the object's (or system's) _____________________.

D. The (–) sign in Hooke's law indicates WHAT?

E. Based on your FBD, write an x-direction, Newton's 2nd law equation. Substitute into it the Hooke's law equation given in Part C, then solve your equation for the mass's acceleration.

F. Into your Part E answer, substitute suitable answers from HW2, P3, Part I. Solve for angular frequency.

G. Use your Part F answer to determine the angular frequency of the mass-spring systems given below.

i. m = 2 kg
k = 32 N/m

ii. m = 24 kg
k = 6 N/m

iii. m = 100 kg
k = 4900 N/m

H. Use an equation you wrote in your Part A answers to determine the period for each of the systems in Part G. Round your answers to two places past the decimal, and include the correct unit.

i.

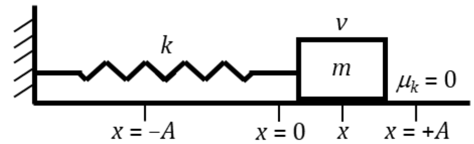
ii.

iii.

I. Combine your Part F answer with the equation you used in completing Part H to derive a (hopefully) recognizable equation for the period of a mass-spring system.

Uδ, HW2, P5
Reference Video: "Simple Harmonic Motion and Energy Conservation"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

The figure at right shows a mass oscillating in SHM. Answer the questions based on the variables shown in the figure.



A. CIRCLE all types of energy the system has at each location, then write an equation expressing the total energy at that point.

i. $x = -A$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=-A} =$

ii. $x = 0$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=0} =$

iii. $x = x$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=x} =$

iv. $x = +A$ ELASTIC POTENTIAL KINETIC Energy equation: $E_{x=+A} =$

B. Given that $\mu_k = 0$, it is a safe assumption that mechanical energy is ______________________ in this system.

C. In terms of k, A, m, and v, derive an expression for the position x where the mass has the speed v.

D. Show that your Part C answer gives you one or more recognizable locations x when $v = 0$.

E. In terms of A, x, m, and k, derive an expression for v when the mass is at location x.

F. Check your work up to this point by plugging your Part D answers into your Part E answer. What result do you obtain?

G. Using your Part E answer...At what location x will v be maximized?

H. Obtain an expression for $|v_{max}|$ by plugging your Part G answer into your Part E answer.

I. Substitute your answer from Part F of HW2, P4 into your Part H answer.

J. Comment on how your Part I answer corresponds to what your HW2, P3, Part I answer has to say about v_{max}.

K. When we maximize the stretch (or compression) of a spring, we also maximize the _______________ force exerted by the spring, in accord with the equation of ______________ law, which is _____ = ______. And for a mass-spring system oscillating in ______________ ___________________ motion, a maximizing of the force above will also go along with a maximizing of the mass's _________________________, according to the equation of _______________ _____ law, which is _____ = ____.