Magnetism

7.1 Magnetism and the Magnetic Field B

You are already familiar with **magnetism**. You know that magnets have two magnetic poles, a north (N) pole and a south (S) pole. A **magnetic pole** is a region where the magnetic field is strongest. You might not know that there is no such thing as a "magnetic monopole"; ALL magnets – always – have two poles. Unlike electric charge, which we can separate and isolate into (+) and (–) entities, N and S poles cannot be separated and isolated; where we find one, we necessarily find the other. Halving a bar magnet (which has one N pole and one S pole) merely creates two smaller bar magnets, each with its own N and S pole. And cutting those new magnets in half...gives the same results. We cannot isolate a single magnetic pole; despite lots of scientific effort, we find no evidence for the existence of "magnetic monopoles."

Similar to the behavior of electric charges, opposite poles on two different magnets attract, and the like poles on two different magnets repel. Like the electric force and gravitational force, the forces magnets exert on each other are inversely proportional to the square of the distance between them; that is, all these forces are **inverse-square laws**.

We've learned already that electric charges q produce electric fields $\boldsymbol{E}$. This is true no matter whether the charge q is in motion…or not. IF the charges happen to be moving, they also bring into being another entity: the **magnetic field**, which is symbolized $\boldsymbol{B}$ and has the unit tesla (T). Once again: Electric charges q that are in motion produce magnetic fields $\boldsymbol{B}$. At any given location, the total magnetic field $\boldsymbol{B}$ at that point is the vector sum of all of the tiny fields $d\boldsymbol{B}$ from every bit of moving charge dq in the vicinity. In other words, magnetic fields $\boldsymbol{B}$ conform to – you guessed it! – the principle of superposition. The magnetic fields of refrigerator magnets and of the Earth itself are due to the unique motion of charges within those objects; unique in the sense that all of the $d\boldsymbol{B}$s do NOT cancel completely…as they do in most objects.

A magnetic field $\boldsymbol{B}$, like an electric field $\boldsymbol{E}$, is a vector field that can store energy U. Electric fields $\boldsymbol{E}$ and magnetic fields $\boldsymbol{B}$ are also similar in the sense that they both exert forces – not on masses, as gravitational fields $\boldsymbol{g}$ do – but on charges q. Just like electric fields $\boldsymbol{E}$ are PRODUCED by both stationary and moving charges, so too can existing $\boldsymbol{E}$ fields (one might call them "external" $\boldsymbol{E}$ fields) EXERT FORCES on both stationary and moving charges.

Magnetic fields $\boldsymbol{B}$ are different. Not only are magnetic fields $\boldsymbol{B}$ produced ONLY by *moving* charges, an existing (or external) $\boldsymbol{B}$ field can exert forces only ON other *moving* charges. That is: Magnetic fields $\boldsymbol{B}$ exert no forces on stationary charges. Perhaps you can see why we needed to fully study electric current I in previous units before delving into the world of magnetism; magnetism has no meaning – it does not exist, and it has no effects – unless charges are moving.

7.2 Magnetic Field Lines

Just like we drew electric field lines to help us visualize the electric field $\boldsymbol{E}$, we can similarly draw **<u>magnetic field lines</u>** to help us visualize the magnetic field $\boldsymbol{B}$. Like $\boldsymbol{E}$ field lines, $\boldsymbol{B}$ field lines that are adjacent to each other never tangle or intersect. You recall that $\boldsymbol{E}$ field lines begin on (+) charges and terminate on (–) charges. Like the trajectory of a home run in baseball, where the ball starts at home plate, follows a (nearly) parabolic path, and ends in the seats beyond the outfield, electric field lines have a definite starting point, a definite path, and a definite stopping point. Magnetic field lines, on the other hand, have no beginning and no end, but rather exhibit a closed-loop structure. An analogy here is that magnetic field lines are like the paths of the planets around the Sun: There is no beginning and ending point, merely a continuous circling-around. All magnetic field lines do exactly that.

And around WHAT do magnetic field lines circle? Moving charges, of course! Here is how you determine the direction of a magnetic field ***B*** around a current-carrying wire using what I call the Thumbkin-Twistin' version of the **right-hand rule**...

Point your right thumb in the direction of conventional (i.e., (+)) current. Rotating the fingers and wrist of your right hand around the thumb, while keeping the thumb in place, gives the direction of the ***B*** field. Try this, and perhaps it will help you understand:

Place your right hand on the table in front of you, with the right thumb pointing straight upward and the other fingers just slightly curved. Here, you are essentially representing that you have a (+) charge q moving upward, along your thumb, traveling away from the floor and toward the ceiling. That charge q might be enclosed within a vertically-oriented wire and moving along with a bunch of other (+) charges, or it might be a lone (+) charge just traveling upward, but – regardless – an upward-pointing thumb indicates one or more upward-moving (+) charges.

Now, keeping your thumb in place, rotate your other fingers to-the-left-and-toward-you, while simultaneously bending your wrist slightly, in the natural way that your hand most easily moves. You should see that your fingers rotate in a counterclockwise (CCW) fashion. Try it a few times to make sure you see your fingers rotating CCW. That is the direction of the magnetic field ***B*** any time you have (+) charge traveling vertically upward. The ***B*** field goes around in a complete circle, without beginning or end. (If you can actually do this with your right hand, you probably need an exorcist…)

Now assume you have a (+) charge traveling downward, away from the ceiling and toward the floor. Flip your right hand over, such that your right thumb extends straight downward, toward the floor. Again, curl your fingers and wrist around your thumb; they should curl clockwise (CW). That is the direction of the magnetic field ***B*** any time you have (+) charge traveling vertically downward.

So, if you know the direction of any (+) charge flow, you can use the Thumbkin-Twistin' right-hand rule to ascertain the direction of the magnetic field ***B*** produced by that charge flow. With a current-carrying wire, for example, you have multiple (many-MANY!) charges, all moving in the same direction. Simply point your right thumb parallel to the wire in the direction the (+) current *I* is flowing, and your curling fingers will tell you in what direction the magnetic field lines circle.

One more note: We know how to use the Thumbkin-Twistin' right-hand rule to find the direction of ***B*** field lines…of which there are MANY that could be drawn around a moving charge. These ***B*** field lines are a series of concentric circles, centered on the moving (+) charge. If you place your right hand on the table again, extending your right thumb upward, you remember that we just discovered that the ***B*** field is directed CCW around your thumb. Now imagine an archer's target lying flat on the table in front of you, with your extended right thumb lining up perfectly with the bull's-eye… The rings on the target give you a good visual picture of the many circular ***B*** field lines that we can draw around the moving (+) charge. Perhaps you will guess that the magnitude of the ***B*** field will be stronger for the smaller circles that are closer to the wire and weaker for the larger circles that are farther from the wire, but that's a story for another section.

7.3 Magnetic Force F_B on a Moving Charge q

For a charge *q* moving at a velocity ***v*** in a magnetic field ***B***, the **magnetic force** F_B is found from:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

The magnitude of this force is: $F_B = qvB \sin\theta$ where θ is the smaller angle formed when the tails of the vectors $\boldsymbol{v}$ and $\boldsymbol{B}$ are aligned.

It should be noted that the $\boldsymbol{B}$ in the above equation is NOT the magnetic field $\boldsymbol{B}$ that is generated BY the moving charge q; rather, it is an external $\boldsymbol{B}$ field, a $\boldsymbol{B}$ field from some other source. Something very similar happened a long time ago with the equation $\boldsymbol{F} = q\,\boldsymbol{E}$… Perhaps you recall that the $\boldsymbol{E}$ in that equation was NOT the electric field $\boldsymbol{E}$ that was set up BY the charge q; rather, it was an external $\boldsymbol{E}$ field, an $\boldsymbol{E}$ field from some other source.

There are four points that can be gathered from the vector equation $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$…

1. The magnetic force $\boldsymbol{F_B}$ that acts on any charge q that is in motion through an external magnetic field $\boldsymbol{B}$ is perpendicular to both the velocity $\boldsymbol{v}$ of the charge q and the direction of the external field $\boldsymbol{B}$. This is standard operating procedure which you should know already for all cross products (also called vector products): The vector produced in a cross-product operation is perpendicular to both of the crossed vectors, and that vector product (which is $\boldsymbol{F_B}$, in this case) is maximized when the component vectors (here, $\boldsymbol{v}$ and $\boldsymbol{B}$) are EXACTLY perpendicular to each other.
2. In a similar vein, a net charge q moving with a velocity $\boldsymbol{v}$ in a magnetic field $\boldsymbol{B}$ experiences ZERO magnetic force $\boldsymbol{F_B}$ if the charge's velocity $\boldsymbol{v}$ and the magnetic field $\boldsymbol{B}$ are either parallel or antiparallel. Recall that the magnitude of any vector product equals the area of the parallelogram having the crossed vectors as two adjacent sides of the parallelogram. If $\boldsymbol{v}$ and $\boldsymbol{B}$ are either parallel or antiparallel, no parallelogram can be defined. Another way of understanding this is by referencing the equation for the magnitude of the magnetic force $\boldsymbol{F_B}$: F_B: $= qvB \sin\theta$ If $\boldsymbol{v}$ and $\boldsymbol{B}$ are parallel, $\theta = 0^o$; if they are antiparallel, $\theta = 180^o$. In either case, $\sin\theta$ will equal zero, and so $\boldsymbol{F_B}$ must also be zero. One more time:

A charge q will experience a nonzero magnetic force $\boldsymbol{F_B}$ ONLY IF it "cuts across" (against the grain, if you will) the $\boldsymbol{B}$ field lines; if it moves parallel to them, it will continue moving in a straight line, because the magnetic force $\boldsymbol{F_B}$ on that charge q will be…zero.

3. The equation also shows that, for a particle to experience a magnetic force $\boldsymbol{F_B}$, it must have both a net charge q and it must be moving with some nonzero velocity $\boldsymbol{v}$. A mass m having ZERO net charge q CANNOT experience a magnetic force $\boldsymbol{F_B}$, no matter how it might be moving. Similarly, even if a particle HAS a net charge q, it CANNOT experience a magnetic force $\boldsymbol{F_B}$ if it is stationary.

4. And finally, because a cross product is involved, the direction of the magnetic force $\boldsymbol{F_B}$ can be found – for (+) charge carriers – by employing some variation of the right-hand rule, any of which give identically good results. (My two favorites, I call the "Bang-bang!" version and the "Pat-pat" version.) These different variations of the right-hand rule, as well as the option of using a left-hand rule for (–) charge carriers, will be fully discussed in class.

If a moving charge q enters a region of space in which there exists BOTH an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$, then the **<u>Lorentz force law</u>** will yield the *combined* electrostatic and magnetic force $\boldsymbol{F}$ (also called the **<u>Lorentz force</u>**) on the moving charge q:

$$\vec{\boldsymbol{F}} \;=\; q\vec{\boldsymbol{E}} \;+\; q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \;=\; q\,(\vec{\boldsymbol{E}} \;+\; \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})$$

Note that both the electric ($q\boldsymbol{E}$) and magnetic ($q\boldsymbol{v}$ x $\boldsymbol{B}$) portions of this equation are vectors. These "force-portions" generally will NOT point in the same direction. (Recall that an electric force on q is always parallel (or antiparallel) to an $\boldsymbol{E}$ field, while a magnetic force on q is always perpendicular to a $\boldsymbol{B}$ field.) So, depending on the respective directions of $\boldsymbol{E}$, $\boldsymbol{v}$, and $\boldsymbol{B}$, as well as the charge of q (whether + or –), the two force-portions could point in any possible direction. Once you've found them, the two force-portions must be combined via vector addition, and so great care must be taken in obtaining the total force $\boldsymbol{F}$ on a charge q when using the Lorentz force law.

A uniform magnetic field ***B***, like a uniform electric field ***E***, is unchanging in both its magnitude and direction. Any uniform field – whether magnetic, electric, or otherwise – is best visualized as a series of straight arrows, all parallel to each other and at a constant spacing.

Suppose now that a charge *q* moves into a region of uniform magnetic field ***B*** such that its initial velocity ***v*** is perpendicular to the ***B*** field. Referring back to the first equation in this section, we see that there will be a magnetic force $\boldsymbol{F_B}$ that begins to act on the charge *q*, and that the direction of this force $\boldsymbol{F_B}$ will be perpendicular to both ***v*** and ***B***. You should recall from your studies of Newtonian mechanics that, whenever a net force acts perpendicular to an object's velocity vector, that force – in essence – becomes a centripetal force, and the object will begin to curve in a circular path. The same is true with our case here: Because ***v*** and $\boldsymbol{F_B}$ are perpendicular, the charge *q* is forced into a circular trajectory, the plane of which is perpendicular to the magnetic field ***B***. We will explain this more in class – as well as the freaking-awesome case of "the corkscrew" – but here, it is enough for you to know that these scenarios provide excellent opportunities to combine the concepts and equations of electromagnetism with those from mechanics.

Two last comments about a charge *q* entering a region of uniform ***B*** field and being bent in a circular path by the resulting magnetic force $\boldsymbol{F_B}$...

1. When the path of a charge *q* is bent in a circular path by the action of the magnetic force $\boldsymbol{F_B}$, the work *W* done by the magnetic force $\boldsymbol{F_B}$ on the particle is ZERO. This fact arises because the charge's incremental displacement – at every point – is perpendicular to the direction of the force $\boldsymbol{F_B}$, i.e., the charge's velocity vector ***v*** is forever tangent to the magnetic force $\boldsymbol{F_B}$. You should have learned in mechanics that whenever a force on an object is at right angles to the object's displacement, that force does ZERO work on the object. We have exactly that situation here.

2. Furthermore, the work-kinetic energy theorem states that the net work *W* done on any particle equals its change in kinetic energy *ΔK*:

$$W = \Delta K$$

In the case here: Because the ***B*** field does ZERO work *W* on any charge *q* (always!), it follows that the charge's kinetic energy *K* CANNOT change, and thus its speed *v* MUST remain constant. Therefore, a ***B*** field can alter the direction of the charge's velocity vector ***v*** (by causing a *radial acceleration*), but it cannot (it WILL not) change the charge's speed *v* because it cannot cause a *tangential acceleration*.

7.4 Magnetic Force F_B on a Straight Conductor

For a straight conductor of length *L*, carrying a current *I*, in a uniform (and external) magnetic field ***B***, the magnetic force $\boldsymbol{F_B}$ is given by:

$$\vec{\boldsymbol{F}}_{\boldsymbol{B}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$$

where the direction of ***L*** is the direction of the current *I*.

The magnitude of this force is: $F_B = ILB \sin\theta$

where θ is the smaller angle between ***L*** and ***B***, and the direction of ***L*** is the direction of (+) charge flow.

It is easily shown that the equation above is derived directly from: $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$

Just for fun (!) you might want to try your hand at proving this.

Let's visualize again a uniform magnetic field ***B***, but now there's a current-carrying wire in the ***B*** field. The equation that applies is: $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$

1. For the wire to experience a nonzero magnetic force $\boldsymbol{F_B}$, we see that $\boldsymbol{L}$ and $\boldsymbol{B}$ must NOT be parallel or antiparallel to each other. If they are, then when we use the equation and take the cross product of $\boldsymbol{L}$ and $\boldsymbol{B}$, we will find that $\boldsymbol{F_B}$ is zero. So if our wire is oriented parallel to the uniform field $\boldsymbol{B}$, it will experience NO magnetic force $\boldsymbol{F_B}$, no matter how much current I is flowing through it, nor in which direction that current I might be flowing.
2. Assuming $\boldsymbol{L}$ and $\boldsymbol{B}$ are not parallel or antiparallel, the resulting nonzero magnetic force $\boldsymbol{F_B}$ will always be perpendicular to both $\boldsymbol{L}$ and $\boldsymbol{B}$. Either the "Bang-bang!" or the "pat-pat" right-hand rules can be used to determine the direction of the magnetic force $\boldsymbol{F_B}$, depending on the relative orientations of $\boldsymbol{L}$ and $\boldsymbol{B}$.
3. The magnetic force $\boldsymbol{F_B}$ will be a maximum when $\boldsymbol{L}$ and $\boldsymbol{B}$ are exactly perpendicular to each other.

One consequence of the $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$ equation is that two parallel, current-carrying wires will exert a magnetic force $\boldsymbol{F_B}$ on each other. In accord with Newton's third law, the two forces (one $\boldsymbol{F_B}$ on each wire, due to the other) constitute an action-reaction pair, with the forces being equal and opposite. Here's conceptually what's going on; we'll call the two parallel wires Wire X and Wire Y…

The current through Wire X creates a $\boldsymbol{B}$ field at the location of Wire Y. That $\boldsymbol{B}$ field, in conjunction with Wire Y's current, causes a magnetic force $\boldsymbol{F_B}$ to be exerted on the moving charges in Wire Y via the above equation. Similarly, the current through Wire Y creates a $\boldsymbol{B}$ field at the location of Wire X. That $\boldsymbol{B}$ field, in conjunction with Wire X's current, causes a magnetic force $\boldsymbol{F_B}$ to be exerted on the moving charges in Wire X, again, via the above equation. These ideas will be discussed more in class, but you should at least know that the force $\boldsymbol{F_B}$ between two parallel wires is attractive if the currents I are in the same (i.e., parallel) direction and repulsive if the currents I are in opposite (i.e., antiparallel) directions.

Here is an analogy that might help you remember the content of the previous paragraph: If two people are working together, in the same direction, for the same ends, they will become closer in their relationship. On the other hand, if two people are in opposition, working against each other, they will soon be pushed farther apart. Do with that as you will.

We've been talking a lot about the equation $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$ for finding the magnetic force $\boldsymbol{F_B}$ on a length L of current-carrying wire. Here is something kind of neat: Suppose you have a curved or bent wire, rather than a straight one, in a uniform ***B*** field. Assuming the wire DOESN'T close on itself in that uniform ***B*** field region (i.e., assuming it doesn't form a complete loop within that ***B*** field), then the magnetic force $\boldsymbol{F_B}$ on it is exactly the same as if it were simply a straight wire of length L that started and ended at the same points where the bent wire starts and ends. Let's try an analogy…

You're looking down at a table, on which lies a map of the Midwest. You put your finger on St. Louis and then move it until you hit – not too far away, mostly to the north but a little to the east – Chicago. You measure the straight-line distance between the two cities, cut a straight wire to that length, and set it on the map, connecting the cities with the wire. Imagine there's a current I flowing through the wire, from St. Louis to Chicago. If there's an external ***B*** field across the Midwest, and if that ***B*** field is pointed in any direction except parallel or antiparallel to the wire, then a magnetic force $\boldsymbol{F_B}$ will be exerted on the wire.

Now, there are any number of ways to get from St. Louis to Chicago, other than a straight line. You could go northwest into Missouri, then straight north into Iowa, before turning east to finally get to Chicago. Or you could cut straight east across southern Illinois, turn north when you hit Indiana, and then bend to the west to get to Chicago. So let's cut and bend wires that stretch from St. Louis to Chicago along THOSE two paths; obviously, the wires are longer than our first wire and have multiple bends in them. But, if the ***B*** field across the Midwest hasn't changed from before, then the magnitude and direction of the magnetic force $\boldsymbol{F_B}$ on either of the bent wires will be EXACTLY what they were on the straight wire. And this would be true for a wire of literally ANY path you took, as long as you started in St. Louis, ended in Chicago, and the entire wire was in the presence of the uniform ***B*** field. The force would be the same, even if you went all the way up to Canada before coming back south. ("Neato, eh?" "Oh, sure, ya…What's 'at aboot?")

The $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$ equation is very relevant to electric motors and generators. A **<u>motor</u>** is a device that converts electrical energy into mechanical energy. In a motor, a current *I* (within a wire, obviously) is driven through a magnetic field ***B*** inside the motor's housing; this gives rise to a magnetic force $\boldsymbol{F_B}$ on the moving charges in the wire. However, because the moving charges are trapped within the wire, not only do the charges move due to the $\boldsymbol{F_B}$, but the entire wire moves. (Usually, it rotates.) That rotating wire is attached to something else (e.g., a fan, a blade, or the bristles of a toothbrush) that are then ALSO forced to rotate…and now you're able to dry your hair or shave or brush your teeth. How cool is that?

A **<u>generator</u>** does the opposite; it converts mechanical energy into electrical energy. With a generator, outside forces ***F*** are applied so as to forcibly move a conducting wire *L* within a magnetic field ***B***, and...*voila*! By science (i.e., as if by magic), a current *I* appears in the wire. That's a generator.

The above is a greatly simplified view of motors and generators, but it serves as an introduction. A complete understanding of motors and generators cannot be achieved without knowing about electromagnetic induction...and that is the subject of Unit 8.

7.5 Ampere's Law

In Section 7.4, we met the equation $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$ which deals with the magnetic force $\boldsymbol{F_B}$ that is impressed on a wire of length *L* carrying a current *I*, where the wire is in the presence of an EXTERNAL magnetic field ***B***. In this section, we will deal with **<u>Ampere's law</u>**, which describes the relationship between an electric current *I* and the magnetic field ***B*** that IT PRODUCES. In format, Ampere's law is somewhat similar to Gauss's law which, you might recall, states how the electric flux Φ_E through any Gaussian surface relates to the net charge q contained within that surface, namely:

$$\varphi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{q}{\epsilon_o}$$

where, as before, the constant of proportionality is the permittivity of free space $\epsilon_o = 8.85\ x\ 10^{-12}\ \frac{C^2}{N \cdot m^2}$

Ampere's law looks like this:

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = \mu_o I$$

μ_o is the **<u>permeability of free space</u>**, and it is equal to 4π x 10^{-7}, and then choose your favorite unit:

$$\frac{T \cdot m}{A} \quad or \quad \frac{N}{A \cdot m} \quad or \quad \frac{N \cdot s}{C \cdot m} \quad or \quad \frac{H}{m}$$

You should recognize everything above except the H in the last unit; it stands for the "henry" (named after American physicist Joseph Henry). The henry is the unit for inductance…which is a topic for Unit 8. Like with Gauss's law, the circle in the middle of the integral symbol means that the integral must be taken around some closed path: For Gauss's law, it is a closed *surface*; for Ampere's law, it is a closed *loop*. And, not that we're going to worry about this, but the above version of Ampere's law is valid only in cases where any ***E*** field that is present is static, i.e., NOT changing.

Anyway, you can see that both Gauss's law and Ampere's law have a dot product on one side of the equation. On the other side, some electrical quantity (q or I) is connected to a universal constant (ε_o or μ_o). Ampere's law, like Gauss's law, is useful in situations that exhibit a high degree of symmetry, such as straight wires, flat plates, and coils of wire like solenoids and toroids...whatever those are.

A **solenoid** is a long, cylindrical coil of wire. Imagine a paper towel tube with a string wrapped around it over and over, starting at one end of the tube and looping…looping…looping in a tight helical path, until the string-loop sequence reaches the other end of the tube. That's the idea behind a solenoid, except of course the string is really an insulated, conducting wire. If a current *I* is established in the coiled wire, a ***B*** field is set up within and around the solenoid. Inside the solenoid, this magnetic field ***B*** is uniform; that is, by analogy, at every location WITHIN the paper towel tube, the magnitude and direction of the ***B*** field are the same. Finally, within a solenoid, the ***B*** field lines run parallel to its axis; the lines exit the solenoid at one end, curve around its exterior, and enter the interior again at the other end. By analogy: The ***B*** field lines are crammed into the paper towel tube like long, straight strands of uncooked spaghetti. But the strands don't terminate; each one bends at the end of the tube, exiting the tube, curving around on the outside to the other end of the tube, and re-attaching to the OTHER end of the SAME strand, weirdly forming a weirdly-continuous weird strand of weird spaghetti. (Recall that all ***B*** field lines are closed paths, like the orbits of planets, having no beginning and no ending.) The ***B*** field in a solenoid can be increased by increasing the current *I* or by increasing the number of loops-per-unit-length of solenoid.

To find the direction of the ***B*** field WITHIN a solenoid (NOT OUTSIDE of it), we use a variation of the Thumbkin-Twistin' right-hand rule. First, recall that our original Thumbkin-Twistin' right-hand rule (see Section 7.2) is used to find the direction of ***B*** field lines around a current-carrying wire: Point your right thumb in the direction of conventional current *I* along the wire, and your right fingers curl in the direction of the "orbiting" ***B*** field lines. The variation on this that is useful for solenoids (and toroids, as can be shown) goes as follows: Curl your right fingers in the direction that conventional current is circling around the center of the tube as current proceeds from one end of the tube toward the other end. The direction your right thumb points (which will be either toward one end of the tube or the other) is the direction that the ***B*** field WITHIN the solenoid points. Let's take a short detour now and discuss how one can use a solenoid to turn an ordinary, unmagnetized bar of iron into an electromagnet.

Suppose you have a circular loop in front of you, with the loop upright so you can see through it. It's like you're in a cruise ship and you're looking out a circular, porthole window. If you can visualize the wall that the porthole is in, that would be helpful as well. Okay: Let's now say that a current *I* is going around the porthole-window loop in a counterclockwise (CCW) direction. This current *I* will, obviously, generate a ***B*** field in and around the window-loop. Using Thumbkin-Twistin', convince yourself that, at the precise location of the window's glass, the ***B*** field lines are coming STRAIGHT toward you. They curve, of course, but convince yourself further that, at all other points on the ship's wall, there are ***B*** field lines directed AWAY FROM you. Now, mentally remove the glass of the window so that there's a nice, circular opening. Open up a cupboard in your passenger cabin and take out a cylindrical "box" of oatmeal. (Why is oatmeal always sold in cylindrical containers? And why do we call them "boxes"? And why is there oatmeal in the cupboards of guests' rooms on this cruise liner? Anyway…) Now, tip the cylinder on its side and insert it into the window so that the lid is INSIDE the ship and the bottom of the cylinder is OUTSIDE the ship. ("Ah, that salty ocean air!") As you stand in front of the window, all you see of the box of oatmeal is the plastic lid, and it appears to you as a perfect circle. Visualize how the ***B*** field lines from OUTSIDE the window are piercing the bottom of the box, traveling through the box towards you, and exiting the box by poking out through the underside of the lid. (Nearly done; just hang on, the next paragraph ends it…)

If the box of oatmeal had instead been an IRON cylinder that we had inserted inside this porthole of CCW current, that iron cylinder would be channeling the ***B*** field lines within the window-space and would turn into an **<u>electromagnet</u>**. Specifically, in our case, the bottom of our iron-oatmeal box (i.e., the part OUTSIDE the ship) would be the south (S) magnetic pole, while the lid of our iron-oatmeal box would be the north (N) magnetic pole. It is important that you know that, INSIDE a magnet, ***B*** field lines are directed from the S pole to the N pole. OUTSIDE a magnet, ***B*** field lines go from N to S. To prove this latter fact to you… Remember the ***B*** field lines poking out from under the (N pole) lid? They curve around, go back through the ship's wall, extend out over the water, and then circle back AGAIN to re-enter the box at the (S pole) bottom! (QED.) And, instead of just a SINGLE porthole-window loop, we could use a SOLENOID with many, MANY loops – with the current of our choice going either CW or CCW in each and every loop – to create an electromagnet with virtually any diameter, length, and strength that we desire. THAT'S an electromagnet ☺. Okay, moving on…

A **toroid** is a solenoid that has been bent around such that the cylinder is turned into a circular loop. So now you have to imagine that the paper towel tube is rubbery; you grab the two ends, bend them until they connect, and glue them together so you have the shape of a hollow donut. The ***B*** field within a toroid is NOT uniform, but it is much stronger than the ***B*** field of a solenoid, for the same amounts of current *I* and coiled wire. This is because, unlike in a solenoid, the ***B*** field lines never leave the interior of a toroid; they simply curve around and around through the interior of the donut-shape. In the analogy, before we formed the donut-shape, we had to cook the spaghetti so as to soften it, and then we cut it at each end of the tube. Then, when we formed the donut-shape with the paper-towel tube, the spaghetti bent into a circular shape along with the tube and, by some miracle, each flexible spaghetti strand re-attached (again!) to the other end of the same strand. Finally, just as there is now NO spaghetti outside the donut, there is similarly NO ***B*** field outside a toroid; the ***B*** field on the exterior of a toroid is…zero.

Recall that, when using Gauss's law, we enclosed charges with Gaussian surfaces, which we imagined being pierced by some number of electric field lines. We multiplied each tiny area element *d**A*** by the component of the ***E*** field that was perpendicular to the surface (i.e., PARALLEL to the *d**A*** arrow-poking-out-of-the-shield vector, you recall!) at that location. Then we summed up all of these products. That sum was the total electric flux Φ_E through the Gaussian surface, and it was numerically equal to the amount of enclosed charge *q* divided by the constant ε_o. (Using Gauss's law is easier to USE than to describe... ☺)

With Ampere's law, we will enclose currents I with "fences" called **Amperian loops** or **Amperian paths**. These currents I will be housed – of course – in wires but, when we draw our current-carrying wires on paper, they will look NOT like straight lines or curved lines, but rather like small circles; essentially…dots. This is because we draw our Amperian paths so we can see them on the paper. And, because our currents I need to be perpendicular to our paths, they need to be going either INTO the paper or OUT OF the paper: Dots! The simplest Amperian path is, of course, a circle, but we will use at least one other shape.

Ampere's law (here it is again)... $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = \mu_o\, I$ ...says that the line integral of $\boldsymbol{B}\, d\boldsymbol{s}$ around any closed path (i.e., any Amperian loop) equals $\mu_o I$ where, as mentioned earlier, μ_o is a constant and I is the total current traveling inside the loop-fence. Specifically, the I in the equation for Ampere's law is the algebraic sum of the currents flowing through our current-dots. The convention is that out-of-the-page currents are (+) and into-the-page-currents are (–) and, if we have both types, then there will be a canceling effect when we compute this sum-of-the-currents.

Another convention (which you might have met before) is that whenever some quantity has a direction of out-of-the-page, it is represented on paper as a small circle with a DOT in its center. (Sometimes we show just the DOT.) An into-the-page quantity is represented as a small circle with an X inside. (Sometimes we show just the X.) During these last two units, we will use these into-the-page/out-of-the-page conventions for several quantities, most notably: running-vertically currents I and running-vertically magnetic fields $\boldsymbol{B}$. We will generally NOT, however, have running-vertically currents I AND running-vertically magnetic fields $\boldsymbol{B}$ in the same problem because, if both I and $\boldsymbol{B}$ are running vertically, no matter WHICH way they're going – both into-the-page, both out-of-the-page, or one of each – then, according to…

$$\vec{\boldsymbol{F}}_{\boldsymbol{B}} = I\vec{\boldsymbol{L}} \times \vec{\boldsymbol{B}}$$

…NO magnetic force $\boldsymbol{F_B}$ will be exerted on those moving charges. And how boring is that?

Looking once more at the equation for Ampere's law, it basically says this: For each little *ds* segment along our Amperian loop-fence, we need to find the product of the magnetic field ***B*** (that is parallel to the loop-fence at that point) and the tiny distance *ds*. We will do this all the way around the loop (one time); we will add all of these ***B*** *ds* products up; and finally we will set them equal to $\mu_0 I$. This will enable us to calculate the magnitude of the magnetic field ***B*** at the location of the Amperian path, IF we've chosen our path wisely, so that the ***B*** is constant and can be pulled out of the integral. Like with Gauss's law, Ampere's law is easiest to use when there is a high degree of symmetry and, again like Gauss's law, the description is worse than the application. So, take a deep breath and relax. Everything is going to be fine!