

**APPC, E & M: Unit C HW 7**

Name: \_\_\_\_\_

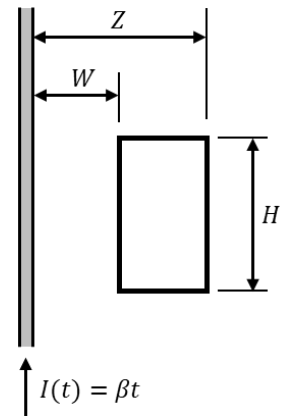
Hr: \_\_\_\_ Due at beg of hr on: \_\_\_\_\_

UC, HW7, P1

Reference Video: "Review of Induction, Faraday's Law, and Lenz's Law (Part IV)"

YouTube, lasseviren1, ELECTROMAGNETIC INDUCTION & FARADAY'S LAW playlist

The figure at right shows a wire that carries a time-varying current and a nearby conducting rectangular loop. The equation for the time-varying current is shown, as are the dimensions of the situation.  $\beta$  is a positive constant.



- A. What must be the units on  $\beta$ ?
- B. As time proceeds, there will be an induced current in the loop. In which direction will this current flow around the loop? (circle)    CW    CCW
- C. We'll now find an equation for the time-varying magnetic flux  $\Phi_B(t)$  through the loop. Begin this by writing the equation for the  $\mathbf{B}$  field magnitude at some radial distance  $r$  away from a wire carrying a current  $I$ . This was basically your answer to Part B of HW4, P5, except here, substitute the expression for  $I$  instead of writing  $I$  itself.

D. Write the general equation for magnetic flux  $\Phi_B$ . This was your answer to Part C of HW4, P5.

E. Rewrite your Part D answer, substituting in your Part C answer in the proper place.

You should have  $\vec{dA}$  elements in your Parts D and E answers. Remember, a  $\vec{dA}$  element represents an area over which the OTHER quantity (that we are multiplying by the  $\vec{dA}$ , in this case  $\mathbf{B}$ ) is constant.

F. Into the figure, draw and label a  $\vec{dA}$  element. Label its thickness as  $dr$  and show that it is a distance  $r$  away from the wire. Check now to make sure that this  $r$  appears in your answers to both Parts C and E.

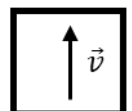
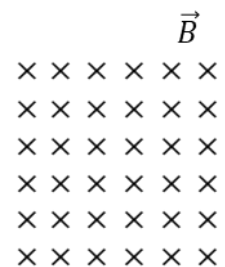
G. It's time to solve what you wrote in Part E so, into your Part E answer, substitute the dimensions of each  $\vec{dA}$  element. Be sure to include the proper integration limits.

H. Solve and simplify your Part G answer to obtain the expression for the time-varying magnetic flux  $\Phi_B$  through the loop.

I. Based on your Part H answer, write an expression for the magnitude of the emf induced in the loop.

J. The last figure shows a conducting loop moving toward a region of uniform  $\mathbf{B}$  field. Circle the correct answer for each quantity.

- |  |                            |                            |                       |
|--|----------------------------|----------------------------|-----------------------|
| i. $I_{ind}$ in the loop, as it enters the $\mathbf{B}$ field          | CW                         | CCW                        | there is no $I_{ind}$ |
| ii. $I_{ind}$ in the loop, as it travels within the $\mathbf{B}$ field | CW                         | CCW                        | there is no $I_{ind}$ |
| iii. $I_{ind}$ in the loop, as it exits the $\mathbf{B}$ field         | CW                         | CCW                        | there is no $I_{ind}$ |
| iv. $F$ on the loop, as it enters the $\mathbf{B}$ field               | <input type="checkbox"/> ↑ | <input type="checkbox"/> ↓ | there is no force     |
| v. $F$ on the loop, as it travels within the $\mathbf{B}$ field        | <input type="checkbox"/> ↑ | <input type="checkbox"/> ↓ | there is no force     |




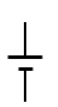

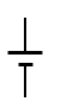

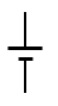

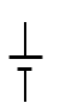

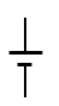

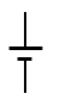
vi.  $F$  on the loop, as it exits the  $B$  field ↑ ↓ there is no force  
 UC, HW7, P2  
 Reference Video: "Basics of Inductance"  
 YouTube, lasseviren1, INDUCTANCE playlist

- A. Inductance is applying \_\_\_\_\_ law and \_\_\_\_\_ law to \_\_\_\_\_.
- B. A solenoid in a circuit is often called an inductor or an \_\_\_\_\_.  
 This device has a property called \_\_\_\_\_.
- C. Draw the schematic symbol for a variable resistor.
- D. At right, draw a schematic of a circuit containing a battery, a variable resistor, and an inductor. Use proper schematic symbols for the battery and variable resistor, but you may use either the schematic or a simple actual representation for the inductor.

With reference to your circuit of Part D...

- E. When we change the resistance, we change the \_\_\_\_\_ in the circuit, which changes the strength of the \_\_\_\_\_ within the inductor and thus the amount of \_\_\_\_\_ through the inductor's loops. This means we will induce a \_\_\_\_\_ (also called an \_\_\_\_\_) in the inductor. This whole process - just described - is called \_\_\_\_\_ - \_\_\_\_\_.
- Basically, what inductors do is they try to prevent the current from \_\_\_\_\_.

F. For each set of conditions, what will be happening with the induced emf in each loop?

	(circle)	(circle)
i.  $I$ increasing		$\mathcal{E}_{\text{ind}} = \text{ZERO}$
ii.  $I$ constant		$\mathcal{E}_{\text{ind}} = \text{ZERO}$
iii.  $I$ decreasing		$\mathcal{E}_{\text{ind}} = \text{ZERO}$
iv.  $I$ constant		$\mathcal{E}_{\text{ind}} = \text{ZERO}$
v.  $I$ increasing		$\mathcal{E}_{\text{ind}} = \text{ZERO}$
vi.  $I$ decreasing		$\mathcal{E}_{\text{ind}} = \text{ZERO}$

G. Finish the right side of each equation, to describe the voltage drop across each type of circuit element.

i.  $V_{\text{resistor}} =$

ii.  $V_{\text{capacitor}} =$

iii.  $V_{\text{inductor}} =$

H. Why is there a (-) sign in your Part Giii answer?

UC, HW7, P3

Reference Videos: (1) "Basics of Inductance (Part II)"  
 (2) "Energy Stored in an Inductor"  
 YouTube, lasseviren1, INDUCTANCE playlist

A. Suppose an inductor has an inductance of 4.0 Henries (H). This means that, at some instant, if the current is changing  $\frac{di}{dt}$  at a rate of 1 A/s, the induced emf  $\mathcal{E}_{\text{ind}}$  in the inductor at that instant is \_\_\_\_\_.

B. For the inductor of Part A, determine – WITHOUT a calculator – the  $\mathcal{E}_{\text{ind}}$  in the inductor for each  $\frac{di}{dt}$ .

i.  $\frac{di}{dt} = 0.50 \text{ A/s}$ ,  $\mathcal{E}_{\text{ind}} =$  \_\_\_\_\_

iv.  $\frac{di}{dt} = 3.0 \text{ A/s}$ ,  $\mathcal{E}_{\text{ind}} =$  \_\_\_\_\_

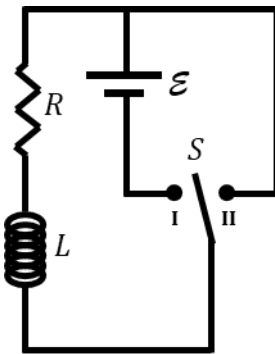
ii.  $\frac{di}{dt} = 0.25 \text{ A/s}$ ,  $\mathcal{E}_{\text{ind}} =$  \_\_\_\_\_

v.  $\frac{di}{dt} = 0.75 \text{ A/s}$ ,  $\mathcal{E}_{\text{ind}} =$  \_\_\_\_\_

iii.  $\frac{di}{dt} = 2.0 \text{ A/s}$ ,  $\mathcal{E}_{\text{ind}} =$  \_\_\_\_\_

vi.  $\frac{di}{dt} = 4.0 \text{ A/s}$ ,  $\mathcal{E}_{\text{ind}} =$  \_\_\_\_\_

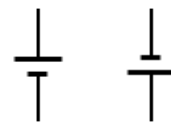
The figure at right shows a circuit consisting of a battery  $\mathcal{E}$ , an inductor  $L$ , a resistor  $R$ , and a switch  $S$ . Points I and II are also indicated.



C. When the switch is thrown to Point I, current will tend to flow around the leftmost loop of the circuit...in which direction?

CW      CCW

D. When the switch is thrown to Point I, there would be an emf induced in the inductor that is directed...how?



E. When the switch is thrown to Point I, the initial magnitude of current around the leftmost loop would be...?

$\mathcal{E}/R$       ZERO

F. A long time after the switch has been thrown to Point I, the magnitude of current around the leftmost loop would be...?

$\mathcal{E}/R$       ZERO

Now, the switch is flipped (essentially instantaneously) to Point II.

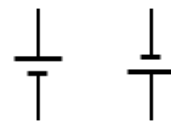
G. When the switch is thrown to Point II, current will flow around the OUTERMOST loop of the circuit...in which direction?

CW      CCW

H. When the switch is thrown to Point II, the initial magnitude of current around the outermost loop of the circuit would be...?

$\mathcal{E}/R$       ZERO

I. When the switch is thrown to Point II, there would be an emf induced in the inductor that is directed...how?



J. A long time after the switch has been thrown to Point II, the magnitude of current around the outermost loop would be...?

$\mathcal{E}/R$       ZERO

K. Write the equation for the potential energy  $U$  stored in an inductor  $L$  that carries a current  $I$ .

L. In AP Physics C, we've met several equations for energy that are similar in format to your Part K answer. Write AT LEAST TWO of those energy equations here.

UC, HW7, P4

Reference Video: "RL Circuit"

YouTube, lasseviren1, INDUCTANCE playlist

Here, we derive an equation for the *rising*, time-varying current  $i(t)$  through an LR circuit, i.e., a circuit having a resistor  $R$  and an inductor  $L$  in series.

A. When the switch is thrown to Point I, write the Kirchoff's Loop

Equation that applies as the current rises. For consistency, let's start at Point I and proceed counterclockwise around the loop. HINT: You will use two of your three answers from Part G of UC, HW7, P2.

B. What is it about your Part A answer that makes it a differential equation?

C. Let's solve your differential equation. First, rearrange your Part A answer so that the  $L$  and  $dt$  terms are the only ones on the left, with  $dt$  in the numerator. HINT: There should be only ONE (-) sign in your answer.

D. We must integrate both sides, so re-write your Part C answer showing that you plan to integrate time from  $t = 0$  to  $t = t$  and current from  $i = 0$  to  $i = i$ .

E. We'll use *u-substitution* to solve your Part D equation. Write here what  $u$  should be set equal to; namely, the right-side denominator of your Part C answer.

F. Now, differentiate your Part E answer with respect to  $i$ , i.e., find  $du/di = ?$

G. Solve your Part F answer for  $di$ . (There should be a (-) sign... ☺)

H. Substitute your answers from Parts E and G into your answer to Part D. Be sure to include the same limits of integration mentioned in Part D.

I. Here, do this: (1) Mentally integrate the LEFT side of your Part H answer (easy!) and (2) move all constants and (-) signs to the left side. Hint: This should leave you with only  $u$ -stuff to integrate on the right. Don't show the steps separately; show only one answer.

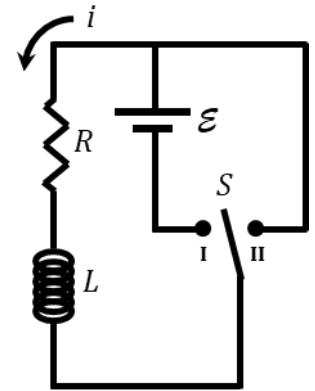
J. Now, do two more steps in one: (1) Integrate the right side of your Part I answer and (2) un-substitute your Part E answer so that there aren't any more  $u$  terms, only  $i$  terms. Be sure to show the integration limits on the far right side of your answer.

K. Two MORE steps in one: (1) Substitute and evaluate the limits of integration on the right side and (2) use a law of logarithms to represent the limits as a quotient of terms, NOT as a difference.

L. Rewrite your Part K answer but, on the right side, eliminate the large fraction line so that the thing within the absolute value sign goes something like: " $1 - ???$ "

M. At this point, in your Part L answer, there should be an  $i$  tucked inside some kind of logarithm function. Take the necessary step to "un-log" the  $i$  term.

N. Solve your Part M answer for  $i(t)$ . DONE!

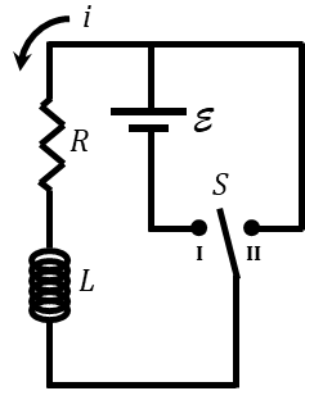


UC, HW7, P5

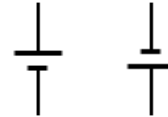
Reference Video: "RL Circuit (Part II)"  
 YouTube, lasseviren1, INDUCTANCE playlist

Here, we derive an equation for the *falling*, time-varying current  $i(t)$  through an LR circuit, by picking up from where the previous problem (P4) left off.

Once the circuit reaches steady state with the switch in the Point I position (this is where P4 leaves off), we instantaneously flip the switch to Point II. At this point, as you can see, the battery will no longer be a part of the circuit.



A. At the instant the switch is thrown to Point II, there would be an emf induced in the inductor that is directed...how?



B. Starting at Point II and traveling CCW, write the Kirchhoff's loop equation for this situation.  
 HINTS: Your answer should end with "= 0"; also, BOTH terms on the equation's left side should have (-) signs in front of them.

C. Manipulate the Part B equation in the following way:  
 Put  $dt$  and  $di$  terms in numerators, have ONLY  $i$ -stuff on the right side, and put the (-) sign on the left.

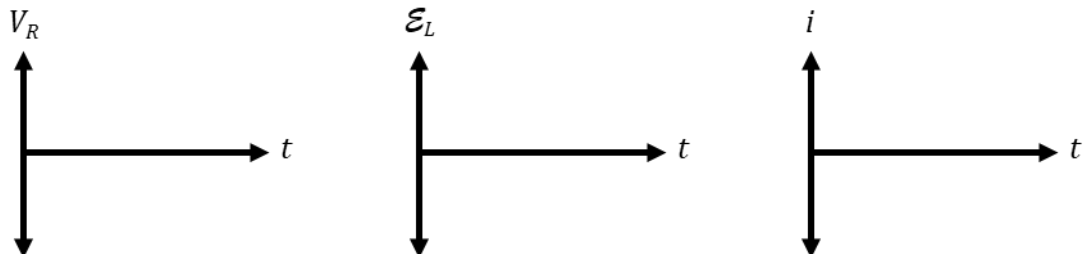
D. Rewrite your Part C equation, showing that you intend to integrate both sides of the equation, with one side being integrated from  $t = 0$  to some later  $t = t$ , and the other side being integrated from the initial  $i = i_0$  to some later  $i = i(t)$ .

E. Now, perform the integration of your Part D answer and simplify using the integration limits you've listed there.

F. There should be an  $\ln$  (with absolute value signs) somewhere in your Part E answer. Perform whatever mathematical operation(s) that are needed to get rid of this  $\ln$ , and then simplify the result to get your final answer, i.e.,  $i(t) = ???$

Summarize HW7, P3-P5 by drawing graphs of resistor voltage  $V_R$ , inductor emf  $\mathcal{E}_L$ , and current  $i$  vs. time  $t$ . If you know the value or expression for a nonzero initial or asymptotic quantity, show this on the graph.

G. For the switch beginning at Point I...



H. For the switch then being flipped to Point II...

