APPC, E & M: Unit C HW 3

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

UC, HW3, P1
Reference Videos: (1) "Ampere's Law (Part I)"
(2) "Introduction to Ampere's Law (Part II)"
YouTube, lasseviren1, SOURCES OF MAGNETIC FIELDS playlist

A. Compare Gauss's and Ampere's laws by writing the correct analog in the Ampere's law column.

	Gauss's law	**Ampere's law**
used to…	calculate ***E***	
employs the use of…	closed surfaces	
uses this constant…named…	ε_o…permittivity of free space	
equation…	$\frac{Q_{enc}}{\varepsilon_o} = \oint \vec{E} \cdot \overrightarrow{dA}$	

The small figure at right depicts a wire carrying a current *I* out of the page.
We wish to find an expression for the magnitude of the magnetic field *B*
at any distance *r*, radially away from the wire.

B. Draw a symbol on the wire, showing that *I* is directed out of the page.

I 

C. We now need an **<u>Amperian loop</u>** around the wire. Because the efficient use of Ampere's law relies heavily on symmetry arguments, the Amperian loop for this case should appear as WHAT shape?

D. With a dashed line, draw your Part C answer and label its only dimension.

E. When we were studying Gauss's law and drew the same shape you drew in your Part D answer, did it represent a "2-D thing" or a "3-D thing"?

F. What about with Ampere's law… Is the shape a "2-D thing" or a "3-D thing"?

<u>IMPORTANT</u>: With an Amperian loop, it is its EDGES that are important, NOT its AREA.
(Look at the equation you wrote in the table above.) With this in mind…

G. Draw and label several tiny $\overrightarrow{dl}$ vectors around the edges of your Part D answer, tangent to the loop.

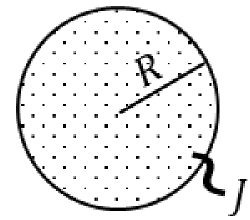
H. Rewrite the equation for Ampere's law.

I. How will the magnitude of ***B*** compare for each of the $\overrightarrow{dl}$ vectors you drew in Part G?

J. Based on your Part I answer, you should now be able to solve the Ampere's law equation to find an expression for *B*.

UC, HW3, P2

Reference Videos: (1) "Applications of Ampere's Law (Part I)"
(2) "Applications of Ampere's Law (Part II)"
YouTube, lasseviren1, SOURCES OF MAGNETIC FIELDS playlist



Here, you will derive equations for the ***B*** field around a thick, current-carrying wire, at various distances from the wire's centerline.

The figure shows a thick wire of radius R. Current flows out-of-the-page.
The uniformly-distributed dots show that the current density J is uniform over the wire's cross-section.

A. Write the equation for uniform current density. (You could refer back to UB, HW3, P2, Part B, if you need a hint.)

First, we'll find an expression for the ***B*** field's magnitude WITHIN the wire, i.e., for $r < R$.

B. Into the figure, with a dashed line, draw an Amperian loop having $r < R$.

C. Write the equation for Ampere's law.

D. The only I relevant to our investigation is the I within (i.e., inside) the Amperian loop of Part B. So now, combine your Parts A and B answers to obtain an expression for the enclosed I, in terms of J and r, i.e., I = ?

E. Ampere's law also requires us to find a path length at the location where we're interested in the ***B*** field. With reference to your Part B answer…Write the formula for the appropriate path length. (You don't need an 'equals' sign; just write the formula.)

F. Use your answers from Parts C, D, and E to solve the Ampere's law equation, i.e., to obtain an expression for B, when $r < R$.

Now, let's find an expression for the ***B*** field's magnitude OUTSIDE the wire, i.e., for $r > R$.

G. Into the figure at the top of the page, with a dashed line, draw an Amperian loop having $r > R$.

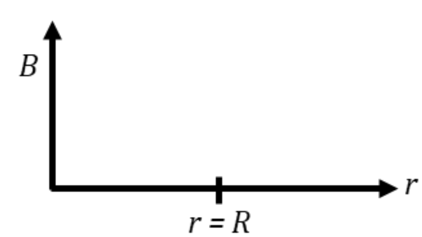
H. Write the Ampere's law equation. (It IS a vector dot product, so don't forget that stuff. 😊)

I. Obtain an expression for the enclosed I for this case.

J. Write the formula for the appropriate path length.

K. Use your answers from Parts H, I, and J to obtain an expression for B, when $r > R$.

L. At right, draw a general graph that supports your answers to Parts F and K. Above each part of the curve, write a proportion that begins… "$B \propto$ …"



M. Describe the DIRECTION of the ***B*** fields that you found magnitudes for in Parts F and K, and mention any rules of thumb (hint, hint!) that you used in figuring this out.

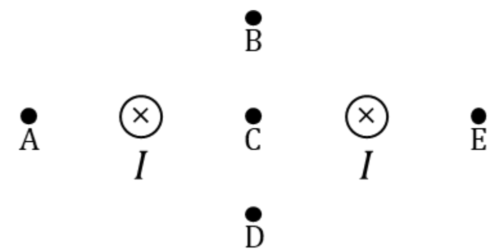
UC, HW3, P3

Reference Video: "Applications of Ampere's Law (Part III)"
YouTube, lasseviren1, SOURCES OF MAGNETIC FIELDS playlist

Each of the two wires in the figure carries an identical current I into the page. The $\boldsymbol{B}$ field due to each, at a specific location, is given by your answer to Part J from UC, HW3, P1. To answer the following questions, you must think about not only the magnitude of each $\boldsymbol{B}$ field contribution, but also its direction.

A. At which location(s) will the $\boldsymbol{B}$ field… (Circle **ALL** correct answers)

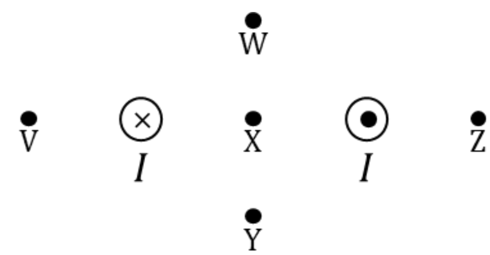
1. …point toward the top of the page? A B C D E none
2. …be equal to zero? A B C D E none
3. …point toward the bottom of the page? A B C D E none
4. …point directly to the left? A B C D E none
5. …point directly to the right? A B C D E none



Now consider these two wires, each of which carries an identical current I, but in opposite directions. The magnitudes of current are identical to Part A, and the locations are also the same, but lettered differently.

B. At which location(s) will the $\boldsymbol{B}$ field… (Circle **ALL** correct answers)

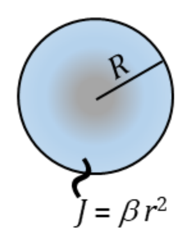
1. …point toward the top of the page? V W X Y Z none
2. …be equal to zero? V W X Y Z none
3. …point toward the bottom of the page? V W X Y Z none
4. …point directly to the left? V W X Y Z none
5. …point directly to the right? V W X Y Z none



C. How do the $\boldsymbol{B}$ field strengths compare? Write either > or < or = in the blanks below.

A ___ E X ___ Y B ___ D V ___ Z A ___ V

The figure shows the cross-section of a thick, current-carrying wire, and we wish to find expressions for $\boldsymbol{B}$ when $r < R$ and also when $r > R$. Unlike in Problem 2 of this HW set, however, the current density J is NOT uniform over the cross-section, but rather is a function of the distance r from the wire's centerline, according to $J(r) = \beta r^2$, where β is a positive constant. We will use Ampere's law, of course, but the trick will be finding the current I that is enclosed within the Amperian loops. Generically, $J = \frac{I}{A}$ and thus $I = JA$, but here, since J is varying, we need to use $I = \int J(r)\, dA$, where dA is the area of an infinitesimally thin ring (equal to $2\pi r\, dr$) and having a "constant" J.



D. Use the above information to derive an expression for the enclosed current I when $r < R$.

E. Use your Part D answer and Ampere's law to find an expression for $\boldsymbol{B}$ when $r < R$.

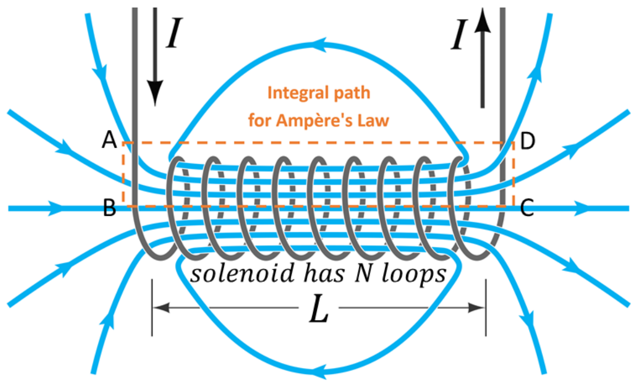
F. Derive an expression for the enclosed current I when $r > R$.

G. Use your Part F answer and Ampere's law to find an expression for $\boldsymbol{B}$ when $r > R$.

UC, HW3, P4

Reference Videos: (1) "Magnetic Field at the Center of a Solenoid (Part I)"
(2) "Magnetic Field at the Center of a Solenoid (Part II)"
YouTube, lasseviren1, SOURCES OF MAGNETIC FIELDS playlist

The figure shows a model of a real solenoid. It shows the magnetic field lines around the device, as well as an Amperian loop that we will use to derive the equation for the ***B*** field inside an **<u>ideal solenoid</u>**. But first, let's review using the right-hand rule (RHR) for solenoids that the narrator discusses in video (2)…



When current flows through the wire wrappings of a solenoid, the solenoid itself becomes a magnet, with one end of the coil becoming the north pole and the other end the south pole.

The RHR for solenoids says: Let the fingers of your right hand point and curl in the direction of the conventional current as it winds around and around the central core of the device. *Your right thumb then points toward the north-pole end of the solenoid*.

A. In the figure, which end of the solenoid is the north pole? (CIRCLE) LEFT RIGHT

We will now derive the equation for the ***B*** field inside an *ideal solenoid* using Ampere's law. Note, in the figure, there is a current *I* in the wire, the solenoid has a length *L*, and there are *N* loops of wire. The Amperian loop is shown in dashed lines, with its four corners labeled (A, B, C, and D). Note also that the coiled wire "pokes through" the "membrane" of the Amperian loop ONE TIME PER WINDING.

B. Write the equation for Ampere's law.

C. Since each of the *N* coils pokes through the membrane of the Amperian loop, and since each loop carries a current *I,* what is the total $I_{enclosed}$ within the Amperian loop shown?

Your Part C answer essentially takes care of the left side of Ampere's law. As we turn our focus to the equation's right side, we recall that, up to this point, all our Amperian loops have been circles. This was so that our ***B*** field would have a constant value for each $\vec{dl}$ segment around the edge of the loop. For any solenoid, however, the ***B*** field CANNOT be constant for any reasonably-shaped Amperian loop. We get around this by using a rectangular loop and assuming an *ideal solenoid*. Unlike the real solenoid in the figure above, an ideal solenoid has ***B*** field lines that continue going in a straight line for a very long *axial* distance (along the solenoid's axis), once they get outside the solenoid. Then, a very long axial distance away, they finally curve back around, "swinging wide" to a very far *radial* distance away, and complete their continuous loops. Also, within the solenoid, the *B* field is constant. So, <u>assuming an ideal solenoid</u>…

…we will "flesh out" the right side of Ampere's law, thusly: $\int_A^B \vec{B} \cdot \vec{dl} + \int_B^C \vec{B} \cdot \vec{dl} + \int_C^D \vec{B} \cdot \vec{dl} + \int_D^A \vec{B} \cdot \vec{dl}$

D. Which TWO of these terms will be zero…because the $\vec{B}$ and $\vec{dl}$ bits are $\perp$?

E. Which ONE of these terms will be zero…because there is NO $\vec{B}$ field along that edge?

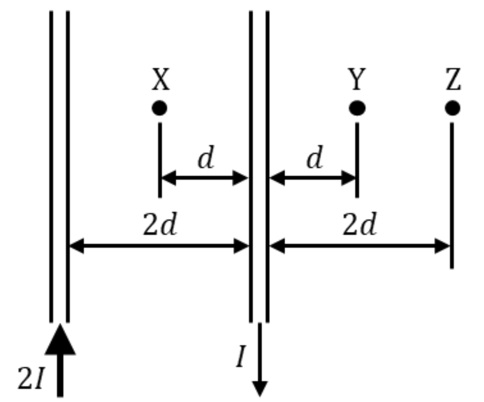
F. Use the one remaining term, along with your Parts B and C answers, to obtain the equation for the ***B*** field inside an ideal solenoid.

UC, HW3, P5

Reference Videos: (1) "Magnetic Field Due to Two Wires"
(2) "Magnetic Field of One Current-Carrying Wire on Another"
YouTube, lasseviren1, SOURCES OF MAGNETIC FIELDS playlist

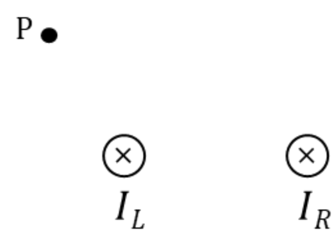
A. To review, write the equation for the ***B*** field magnitude at some radial distance r away from a wire carrying a current I. You derived this in Part K of UC, HW3, P1.

B. Use your Part A answer to write expressions for the ***B*** field at each lettered location, in terms of the given variables. Note that the ***B***-field contributions from each wire could be additive OR subtractive. STATE ALSO each nonzero B field's direction.



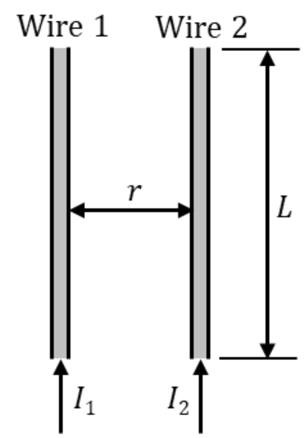
- X:
- Y:
- Z:

C. The next figure shows two wires (Left and Right) carrying equal currents into the page. At Point P in the figure, draw two vectors that indicate both the exact direction AND approximate magnitude of the ***B***-field contribution from each wire. Label your vectors (each of which should originate on Point P) as $\vec{B}_L$ and $\vec{B}_R$.



D. In the space to the right of the previous figure, make a freehand sketch of your $\vec{B}_L$ and $\vec{B}_R$ vectors adding together graphically to show the actual ***B*** field at P. Also, draw in the resultant and label it $\vec{B}_{res}$.

The next figure shows two wires carrying different currents in the same direction. Here, we will investigate the interplay between moving charges that *generate* ***B*** fields and moving charges *having magnetic forces exerted on them by (other)* ***B*** fields.



E. Here (____), draw the symbol that shows the direction of the ***B*** field generated by I_1 at the location of Wire 2. This symbol applies along the entire length of Wire 2.

F. Combine your Part E answer, the known direction of I_2, and the RHR of your choice to figure out the direction of the magnetic force $\boldsymbol{F_B}$ on Wire 2's moving charges. Draw a symbol here (____) that indicates that direction.

G. Here (____), draw the symbol that shows the direction of the ***B*** field generated by I_2 at the location of Wire 1. This symbol applies along the entire length of Wire 1.

H. Combine your Part G answer, the direction of I_1, and a RHR to determine the direction of the magnetic force $\boldsymbol{F_B}$ on Wire 1's moving charges. Draw that directional symbol here (____).

I. From your Parts F and H answers, we see that parallel wires conducting currents in the same direction exert what force on each other? ATTRACTIVE REPULSIVE

J. What if the parallel wires had oppositely-flowing currents? ATTRACTIVE REPULSIVE