Current, Resistance, and Power

5.1 Current I, Resistivity ρ, and Resistance R

Electric current I is the rate at which moving charge passes a fixed point.

$$I = \frac{dq}{dt}$$

The unit for current I is amperes, or amps (A), where 1 A equals 1 C of charge passing a fixed point in 1 s. While it does have both a magnitude and a direction, current I is a scalar quantity, NOT a vector quantity. This is because, unlike vector quantities such as fields and forces, current I does NOT obey the law of vector addition, which involves the relationship between component vectors and resultant vectors. In short: Current is a scalar because you can't Pythagorize it. If you COULD Pythagorize it, then when you DID Pythagorize it – all the way around a complete circuit – you'd get…zero. By convention, the direction of current I is the direction in which (+) charge carriers would move.

Current I in a conducting wire results when an electric field $\boldsymbol{E}$ is established within the wire, directed parallel to the wire. The $\boldsymbol{E}$ field is channeled along and through the wire and has the direction of going away from regions of higher potential V and toward regions of lower potential V. Thus, when a battery is causing the current, the $\boldsymbol{E}$ field within the wires, and thus the conventional (+) current, is directed away from the (+) terminal – which has the higher potential V – and toward the (–) terminal, which has the lower potential V. Said again: Through a circuit's non-battery components, the current I and local electric field $\boldsymbol{E}$ always have the same direction: from high V to low V.

As you might have gathered from the above, for an electric current I to flow, we must connect together two regions of differing potential V using some kind of conducting material. The key to maintaining a

CONTINUALLY-FLOWING current I is that the two regions of differing potential V must be able to maintain their potential difference ΔV over an extended time period. A battery maintains this ΔV by taking charges at its low-potential terminal (i.e., the – one) and adding energy to them while, at the same time, moving them to its high-potential terminal (i.e., the + one). It is because batteries maintain this constant ΔV that current I can continually flow in battery-powered circuits. Capacitors, on the other hand, CANNOT maintain a constant potential difference ΔV between their plates; charge flows only until the ΔV between plates reaches zero.

Resistivity ρ is a material property that depends on what substance we have. For example, copper and lead have different resistivities. The resistance R of a resistor depends not only on its component material's resistivity ρ, but also on the geometry of the resistor. For a resistor consisting of a mass of material having resistivity ρ, a uniform cross-sectional area A, and a length l, the resistance R is given by:

$$R = \frac{\rho\, l}{A}$$

While you should have learned in previous physics studies that wires don't have much resistance, they do have some (as long as they aren't superconductive). The above equation allows us to easily calculate the resistance R of a uniform wire. For a wire, the cross-sectional area A (which, you note, is in the denominator) will be a REALLY tiny number; and since wires don't have very big resistance R values, that must mean that the resistivity ρ is a SUPER tiny number. This is, indeed, the case, as the ρ values for metals are in the range of 10^{-7} or 10^{-8} Ω-m (ohm-meters).

Once again: The difference between resistivity ρ and resistance R is that resistivity ρ is a property of a given MATERIAL; resistance R is a property of a given OBJECT. The object, of course, is made of a given material, but it also embodies a certain geometry or construction that affects its resistance R.

5.2 Ohm's Law

Formally, **Ohm's law** states that the current I flowing through a resistor equals the potential difference ΔV across the resistor divided by the resistance R:

$$I = \frac{\Delta V}{R}$$

Informally, Ohm's law is often represented in its "no denominators" form: $\Delta V = I\,R$

Perhaps you learned in an earlier physics course that Ohm's law applies not only to circuits as a whole, but also to any portion of a circuit. For example, plugging the ΔV across the circuit (say, across the battery) and the resistance R of the entire circuit into Ohm's law will yield the total current I flowing through the circuit (which is also the current I flowing through the battery). Plugging the ΔV across a specific circuit element (say, a resistor) and the resistance R of the resistor into Ohm's law will yield the current I flowing through the resistor. Said again: Ohm's law applies to a whole circuit…or any part of it.

An **ohmic conductor** (or **ohmic resistor**) is one for which the conductor's resistance R is independent of the applied potential difference ΔV across it; that is, for an ohmic conductor, the potential difference ΔV and the current I are directly proportional, with the constant of proportionality being the resistance R:

$$R = \frac{\Delta V}{I}$$

You see that the above equation is simply a re-write of Ohm's law, solved for resistance R.

On a graph, with current I plotted along the horizontal axis and potential difference ΔV plotted along the vertical, the slope of the graph is the resistance R. You can see how the above equation supports this claim, where essentially…

$$slope = R = \frac{RISE}{RUN} = \frac{\Delta V}{I}$$

Thus, on such a graph, a steeper slope indicates a higher resistance. It goes without saying that there can be no such thing as a negative resistance *R*; such a case would violate the law of conservation of energy.

For a **non-ohmic conductor** (or **non-ohmic resistor**), the ΔV-to-I ratio is not constant. For such resistors, current *I* will indeed increase as potential difference ΔV is made to increase, but it has been observed that equal incremental increases in *I* require proportionately larger and larger increases in ΔV. A ΔV-vs.-I graph of a non-ohmic resistor (remember, current *I* is on the *x*-axis) would be a curve that becomes increasingly steep as we move to the right, which shows that a non-ohmic resistor's resistance *R* increases with increasing applied potential difference ΔV. A ΔV-vs.-I graph of an ohmic resistor, on the other hand, is a line of constant slope; this is consistent with the behavior of an ohmic resistor, whose resistance *R* remains constant, regardless of the potential difference ΔV applied across it.

For this class, unless explicitly told otherwise, you are to assume that all resistors are ohmic resistors.

Superconductors are materials that lose all resistance at low temperatures. Many metals and ceramics become superconductive at temperatures near that of liquid nitrogen (-196^{o}C). The ultimate, not-yet-attained goal for scientists is to engineer materials that are superconductive at room temperature. So, feel free to go ahead and do that, someday. (Thanks very much.)

5.3 Current Density J

The current *I* in a conductor is also related to the density of charge carriers *N* in the conducting material, the charge *e* on each carrier, the soon-to-be-explained drift speed v_d, and the cross-sectional area *A* of the conductor via the equation:

$$I = Nev_d A$$

The density of charge carriers *N* (unit = carriers/m^3) depends on the conductive material. A metal will have a higher value of *N* due to its relatively large number of conduction electrons, a **semiconductor** such as silicon (Si) or germanium (Ge) will have a lower value of *N* due to its relatively fewer number of conduction electrons, and an insulator will have an *N* value of essentially…zero. Semiconductors, although they are poor conductors in their pure state, can successfully conduct if they are doped with certain impurities, i.e., other atoms that contribute charge carriers to the semiconducting material.

The charge *e* is the elementary charge, i.e., the charge on an electron or proton: e = 1.60 x 10^{-19} C. Recall that conventional current *I* is the flow of (+) charge carriers; thus, the (+) nature of the constant *e*. So, although it is the (–) charge carriers (i.e., the conduction electrons) that actually move in an electrical circuit, and although they of course move opposite to the way any (+) charge carriers would flow, it has been found that, for nearly all circumstances, there is no discernable difference between real (–) charge carriers traveling THIS way and imagined (+) charge carriers traveling THAT way. Therefore, the convention (which is that the (+) charge carriers are the ones doing the traveling) remains in place.

For conductors in equilibrium, the conduction electrons are in a state of constant, random motion, at speeds in the vicinity of 1 x 10^6 m/s. However, there is no NET flow of these charge carriers in any particular direction, and thus…no current. Things change when an electric field ***E*** is imposed through the conductor. The ***E*** field passes through the conductor at nearly the speed of light; charge carriers (i.e., the conduction electrons) are affected by this imposed ***E*** field (remember the equation $\boldsymbol{F} = q\,\boldsymbol{E}$?); and a current *I* begins to flow. The charge carriers still have speeds of around 1 x 10^6 m/s; their motion is still random…but it is a tiny bit LESS random than before. The result is that the conduction electrons begin to migrate – AGAINST the ***E*** field, of course – at a very low average speed called the **drift speed** v_d, typically around 0.01-0.1 mm/s. This drift speed v_d is very low because, although the conduction electrons are all being pushed due to the imposed ***E*** field, they are having to fight their way through a dense crowd of other conduction electrons, atomic nuclei, and swarms of kernel electrons. And real (–) charge carriers having a drift velocity v_d in THIS direction is exactly equivalent to (+) charge carriers having a drift velocity v_d in the OTHER direction…which is why our convention of (+) charge flow works just fine.

The magnitude of the average **current density** *J* through a conductor is the current *I* per unit cross-sectional area of the conductor *A*:

$$J = \frac{I}{A} = Nev_d$$

Current *I* is in amperes (A); area *A* is in square meters (m^2); thus, current density ***J*** (which is technically a vector, more on this in a minute) has the unit A/m^2.

If a conducting material has different cross-sectional areas *A* at different points along its length, the average current density ***J*** will differ from point to point, but the total current *I* will be the same at each location. That is: Suppose you have a circuit in which 3 A of current are flowing through each component. Suppose, too, that the circuit is constructed with a wire having different thicknesses at different points in the circuit. The current *I* will be the same at all locations, but the regions where the wire is thicker will have a SMALLER average current density ***J*** (because of their larger cross-sections *A*), while the regions of thinner wire will have a LARGER average current density ***J*** (because of their smaller cross-sections *A*).

The previous equation assumes that the current density ***J*** over an entire cross-section is uniform. Suppose, however, that the current density ***J*** is non-uniform over a given cross-section. If ***J*** is different for each little ***dA*** element of the cross-section, we could represent this variation in ***J*** as a function of location on a cross-section by writing it as $\boldsymbol{J}(x, y)$. In such a case, the total current I through the cross-section would be:

$$I = \oint \vec{\boldsymbol{J}}(x, y) \cdot \overrightarrow{d\boldsymbol{A}}$$

You see that we have represented the current density $\boldsymbol{J}$ as a vector quantity, which points in the direction of (+) charge flow (or in the direction of the imposed $\boldsymbol{E}$ field, take your pick). This quantity is dotted with the familiar $d\boldsymbol{A}$ vector (which points in the same direction) to yield the scalar quantity of current I.

In an ohmic conductor, the current density vector $\boldsymbol{J}$ depends on the electric field vector $\boldsymbol{E}$ through the conductor and the conductor's **<u>conductivity</u>** σ:

$$\vec{\boldsymbol{J}} = \sigma \vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{E}}}{\rho} \qquad or \qquad \vec{\boldsymbol{E}} = \rho \vec{\boldsymbol{J}} = \frac{\vec{\boldsymbol{J}}}{\sigma}$$

It can be inferred from the above equations that conductivity σ is the inverse of resistivity ρ. Resistivity ρ has the unit ohm-meter (Ω-m); the official SI unit for conductivity σ is the siemen (S), which was formerly called (and sometimes still is) the mho or 1/ohm-meter (1/Ω-m).

5.4 Power P

If a potential difference ΔV is maintained across a circuit element, the **<u>power</u>** P is equal to the product of the current I and the potential difference ΔV:

$$P = I \, \Delta V$$

Recall that power P can be variously described as the rate at which work is done, the rate at which energy is transferred from Point A to Point B, or the rate at which energy is transformed from one form into one or more other forms. For electrical circuits, power P is the rate at which energy is supplied to a circuit element. If the energy is delivered to a resistor (as opposed to a different circuit element, such as an electric motor), that energy appears in the form of internal energy in the resistor. This internal energy is manifested as a heating up of the resistor, as the charge carriers and the atoms of the resistor collide. Combining the power equation with Ohm's law, it can be shown that the internal energy in a resistor is consumed (and dissipated as heat) at the following rate:

$$P = I^2R = \frac{(\Delta V)^2}{R}$$

The heat losses associated with the above equation are variously called Joule heating, Ohmic heating, resistive heating, or I^2R losses (pronounced "I-squared-R losses"). This is trivia; you need not memorize any of these terms.