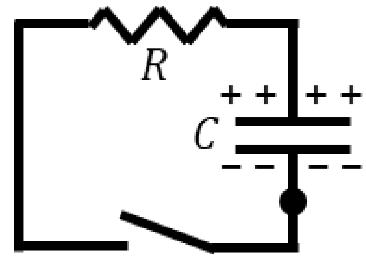
APPC, E & M: Unit B HW 6

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

UB, HW6, P1
Reference Video: "Discharging a Capacitor (RC Circuits)"
YouTube, lasseviren1, AIR RESISTANCE AND RC CIRCUITS playlist

Here, you derive the equation for the charge as a function of time on the plates of a discharging capacitor.

The capacitor C has already been charged, let us say, such that the initial charge on the capacitor is Q, with the plates having equal and opposite amounts of charge, as shown in the figure. There is also a resistor of resistance R in the circuit but, as you can see, there is no longer a battery.



A. When the switch is thrown, a time-varying current i (or $i(t)$, if you wish) will begin to flow in the circuit. Into the figure, draw a curved arrow in the proper direction to represent this time-varying current. Label the arrow by writing next to it something that indicates the quantity it represents.

B. Starting at the dot and traveling CCW, write the Kirchhoff's loop equation for the capacitor discharging (which, obviously, is when the current is flowing). Your equation should end with " = 0 " and the variables that should appear are R, C, q, and i. (q and i are lowercase because they will vary with time.) If you need a partial hint, refer back to HW5, Problem 5, Part A(2).

C. Refer back now to HW5, Problem 5, Part A(1), and then rewrite accordingly your Part B answer above. You should now have a differential equation.

D. Circle the correct answers:

For a discharging capacitor, the quantity dq/dt is… INCREASING DECREASING

…and so, at any instant, dq/dt is a quantity that is… POSITIVE NEGATIVE

E. Combine your Parts C and D answers to write your equation yet again; this time, there should be no (–) signs.

F. Now we are ready to solve your equation of Part E. First, put the quantity with the dt on the other side of the equation, such that the " = 0 " disappears.

G. Manipulate your Part F equation so that both dt and dq end up in numerators. (So we all stay on the same page, keep the (–) sign with the dt.)

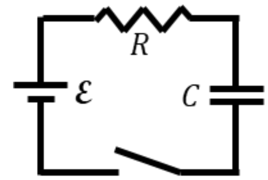
H. Rewrite your Part G equation, showing that you intend to integrate both sides of the equation, with one side being integrated from $t = 0$ to some later $t = t$, and the other side being integrated from the initial $q = Q$ to some later $q = q$.

I. Now, perform the integration of your Part H answer.

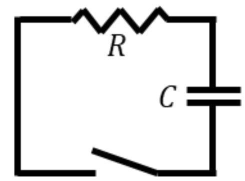
J. There should be an *ln* somewhere in your Part I answer. Perform whatever mathematical operation(s) needed to get rid of this *ln*, and then simplify the result to get your final answer, i.e., $q(t)$ = ???

UB, HW6, P2

Reference Video: "The Time Constant for an RC Circuit"
YouTube, lasseviren1, AIR RESISTANCE AND RC CIRCUITS playlist



CHARGING

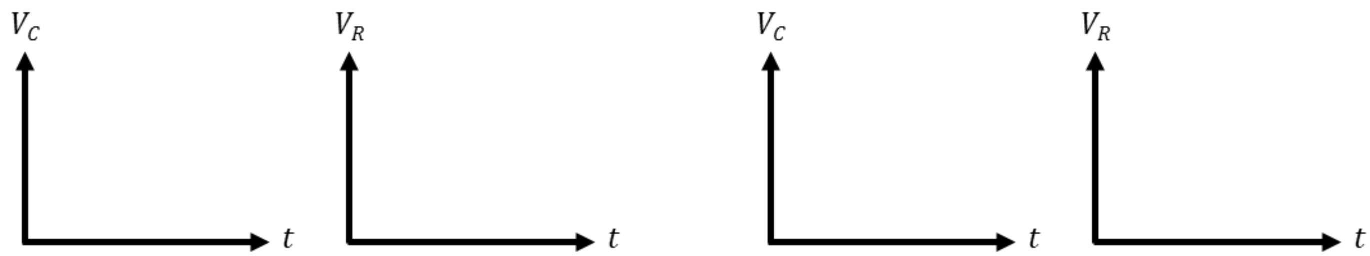


DISCHARGING

Back in HW5, Problem 4, you drew graphs for how the charge q, current i, and voltage across the capacitor V varied with time, during both charging and discharging. Here, we will make ourselves more aware of the effect of a resistor R in the circuit. Sample charging and discharging circuit diagrams are shown at right.

Here, we will first distinguish between the *voltage across the capacitor* V_C and the voltage across the *resistor* V_R .

A. Draw voltage-vs.-time graphs for charging… …and discharging.



B. Now, as you did in HW5, Problem 4, Part C…Below EACH graph, write TWO of the following four labels:

1 – e^–? **decaying** **e^–?** **climbing**

For circuits with both a resistor R and a capacitor C, the exponent given above (i.e., the –?) is $-t/RC$, and the quantity RC is called the **time constant** τ; that is, $\tau = RC$. So the exponential parts of the time-dependent-behavior-equations for capacitors are often written:

$1 - e^{-t/\tau}$ AND $e^{-t/\tau}$

C. From the equation above, you can see that, if you know the values of R and C, then τ is VERY easy to calculate. Furthermore, it is important to know the value of τ because it is analagous to another time constant, one you learned about in chemistry, the one for radioactivity: ____________________.

D. By what decimal (< 1.0) does a radioactive substance change during ONE of your Part C answers? _____

E. What about during TWO of your Part C answers? (Write your work out.) _____ x _____ = _____

F. The same kind of idea holds for the time constant τ for RC circuits, except that the decimal is different from your Part D answer. When the time t = τ, to two decimal places, what do the following simplify to? $1 - e^{-t/\tau}$ = _____ $e^{-t/\tau}$ = _____

G. Below the appropriate Part F expressions, write either **decaying** or **climbing** .

H. How about during TWO Part F answers? Fill in the blanks; there's no need to calculate a final decimal.

TWO of $(1 - e^{-t/\tau})$ = _____ x _____ TWO of $(e^{-t/\tau})$ = _____ x _____

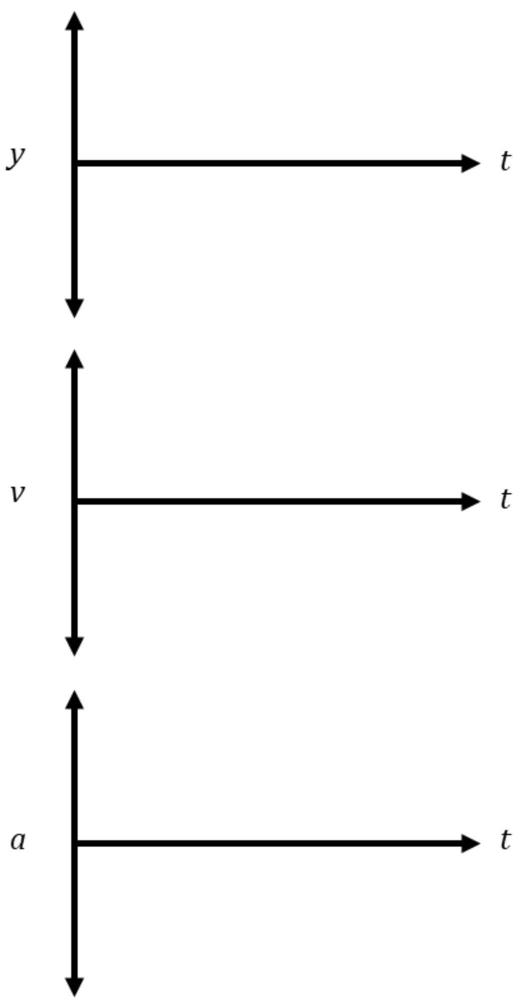
UB, HW6, P3

Reference Video: "Review of Unit on Air Resistance & RC Circuits (Growth & Decay in Physics, Part I)" YouTube, lasseviren1, AIR RESISTANCE AND RC CIRCUITS playlist

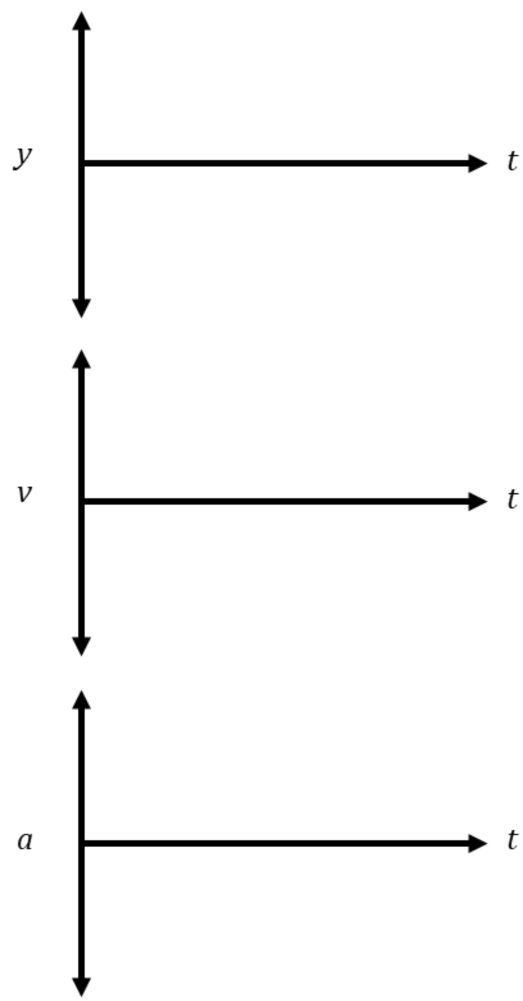
In this problem, we will compare the motion of objects that are NOT experiencing friction (or air resistance which, by the way, IS friction-with-the-air) with objects that ARE. Let's consider an object of mass m, dropped from rest at an elevation near Earth's surface, where the acceleration due to gravity is g. Draw graphs below, using the following criteria:

- Assume that the initial position will be $y = 0$, with the y-values becoming more (–) with time t.
- Where possible, designate known values along a vertical axis, such as the values of asymptotes.
- If a slope is ZERO, then draw an arrow pointing at that region and write, "slope = zero".
- If a slope is NON-ZERO but CONSTANT and you know what its value is (like g), then draw an almost-triangle (you know, a rise-over-run thing; basically an "L" shape) in that region and write the value of the slope (like g!).

A. NO air resistance



B. YES air resistance (and where $F \propto v$)



C. Back in HW5 Problem 3, you solved a differential equation for the case of Part B above. Along the way to doing so, a natural logarithm (ln) appeared in your work. In this video, the narrator almost always uses absolute value symbols any time he writes a natural logarithm, e.g., instead of writing… $\ln v$, he writes… $\ln|v|$. Why? In other words, what is true about logarithms that would cause the narrator to (usually, anyway) write $\ln|v|$?

UB, HW6, P4

Reference Video: "Review of Unit on Air Resistance & RC Circuits (Growth & Decay in Physics, Part II)" YouTube, lasseviren1, AIR RESISTANCE AND RC CIRCUITS playlist

Here, you will do some calculations involving *RC* circuits, some of which involve the time constant τ.

A. We often talk about what is happening in a system at specific times, say, $t = 0$ or $t = \infty$. In an *RC* circuit, besides $t = 0$ or $t = \infty$, we might be interested in what is happening at $t = \tau$ or $t = 2\tau$, etc. In this video, the narrator gives us an idea of roughly how many τ we need before we essentially reach $t = \infty$. About how many τ are needed to reach this point?

B. With reference to the figures shown on the bottom half of this page, what is the value of τ for this circuit? Include the correct unit.

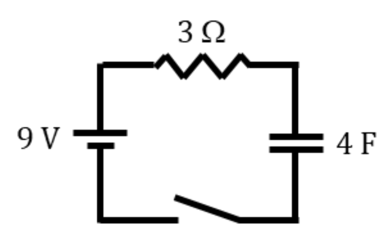
C. Before we go further… To review, during capacitor charging:

- at the instant the switch is thrown, the capacitor acts like a(n)…
- a very long time after the switch is thrown, the capacitor acts like a(n)…

Now complete the tables below, computing the τ and 2τ values to TWO places past the decimal. V_C is the voltage across the capacitor; V_R is the voltage across the resistor; q is the charge on the capacitor plates; i is the current through the resistor (or the battery; same thing, for this circuit); and P_R is the instantaneous power in the resistor (i.e., the rate at which electrical energy is being converted into thermal energy in the resistor). Include the correct unit on each answer.

D. Charging → NOTE: The switch is thrown, i.e., closed, i.e., charging begins, at $t = 0$ s.

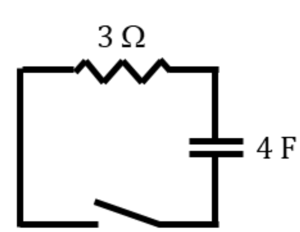
t (s)	0	τ	2τ	∞
V_C				
V_R				
q				
i				
P_R				



CHARGING

E. Discharging → NOTE: The switch is thrown, i.e., closed, i.e., discharging begins, at a re-defined $t = 0$ s.

t (s)	0	τ	2τ	∞
V_C				
V_R				
q				
i				
P_R				



DISCHARGING

UB, HW6, P5
Reference Video: "Review of Unit on Air Resistance & RC Circuits (Growth & Decay in Physics, Part III)" YouTube, lasseviren1, AIR RESISTANCE AND RC CIRCUITS playlist

In this problem, we deal first with the power in a resistor, and how it is related to the thermal energy that is dissipated in the resistor over a period of time.

A. In physics (i.e., in mechanics, or electricity, or any other branch of physics), the unit for power is…

B. Your Part A answer is equivalent to a quotient of two other units. Write that quotient-of-two-other-units here.

C. The unit in the numerator of your Part B answer applies to two key quantities in physics. List those two key quantities here.

D. The unit in the denominator of your Part B answer is for what quantity? (Wow, if you miss this…)

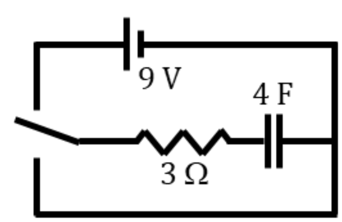
E. Of the two quantities in your Part C answer, which one are we talking about when we notice that a resistor in a circuit is starting to get warm?

F. So now, let's look at three quantities that relate to electric circuits: power, your Part D answer, and your Part E answer. Suppose you wanted to make a something-vs-time graph, such that one of these quantities was along each axis and the slope was the third quantity. Which quantity would be the…

…*x*-axis? …*y*-axis? …slope?

G. What about a something-vs-time graph where the area-under-the-curve was the third quantity? In that case, which quantity would be the…

…*x*-axis? …*y*-axis? …area?



Refer to the figure at right. Look back at the two figures of P4, and note how this circuit is equivalent to BOTH of those. If the switch is UP, it is the charging circuit of P4; if DOWN, it is the discharging circuit. <u>Use numerical quantities derived from the figure at right to answer the following questions</u>.

H. Write here the time-dependent equation for current, as the circuit is charging, i.e., write $i(t) = ?$

I. The equation for the rate at which thermal energy is dissipated (i.e., the power expended) by a resistor is $P = I^2R$. Use this equation, your Part H answer, and the R value given in the figure to obtain the time-dependent equation for power, $P(t) = ?$

J. What calculus operation must you perform on your Part I answer if you wish to obtain the total thermal energy dissipated by the resistor during charging, from $t = 0$ to $t = \infty$?

K. Perform now the operation you mentioned in your Part J answer in order to find the total thermal energy dissipated by the resistor during charging. Include the correct unit on your answer.