

# APPC, E & M: Unit B HW 3

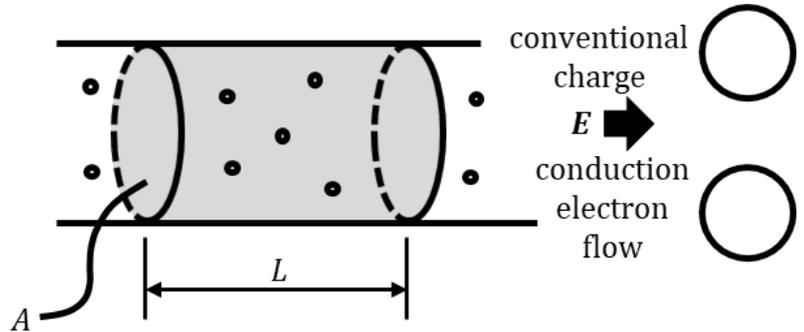
Name: \_\_\_\_\_

Hr: \_\_\_\_ Due at beg of hr on: \_\_\_\_\_

UB, HW3, P1

Reference Video: "Current, Drift Velocity, and Current Density"  
 YouTube, lasseviren1, DC CIRCUITS playlist

The figure shows a section of metal wire of length  $L$  and cross-sectional area  $A$ . Conventional charges (each having the elementary charge  $e$ ) are also shown. The charge density (i.e., the number of charge carriers per cubic meter) of the wire is given by  $N$ . An electric field  $E$  has been imposed through the wire by connecting the wire into a circuit that is powered by a battery. The direction of the  $E$  field is shown by the thick, solid arrow.



- A. Which side of the wire is connected to (or closest to) the... ..(+) battery terminal? LEFT RIGHT  
 ...(-) battery terminal? LEFT RIGHT
- B. In the circle in the upper-right corner, draw an arrow showing the direction conventional charge flows. Into the other circle, draw an arrow showing the direction that the conduction electrons flow.
- C. Because of the huge number of atoms and the swarms of nonconduction electrons in the metal, the conduction electrons cannot move unimpeded through the metal lattice in response to the imposed  $E$  field. They bounce around *mostly* randomly but, over time, they drift in the direction you indicated in your Part B response, with a particular drift velocity  $v_d$ . (This motion is indistinguishable from conventional charge drifting in the opposite direction.) So, in a certain time  $t$ , all of the conventional charges that started within the shaded portion of the wire will have moved out of the shaded region. Write an equation for drift velocity  $v_d$  based on the distance shown in the figure and this amount of time  $t$ .
- D. Solve your Part C answer for time  $t$ .
- E. The total amount of conventional charge that passes out of (or into) the shaded region in the time  $t$  is the charge  $Q$  contained within that region's volume. Write an equation for this amount of charge, i.e.,  $Q =$ . (Hint: There will be four variables on the right side of the equation.)
- F. The current  $I$  through the wire is defined as "charge per time." Write an equation that expresses this definition.
- G. Solve your Part F answer for time  $t$ .
- H. Use the transitive property to equate your Parts D and G answers.
- I. Substitute your Part E answer into your Part H answer.
- J. Simplify and rearrange your Part I answer to obtain an expression for the current  $I$  in terms of the drift velocity  $v_d$ .

UB, HW3, P2

Reference Video: "Resistivity, Resistance, and Conductivity (Part I)"  
YouTube, lasseviren1, DC CIRCUITS playlist

This problem begins as a continuation of Problem 1.

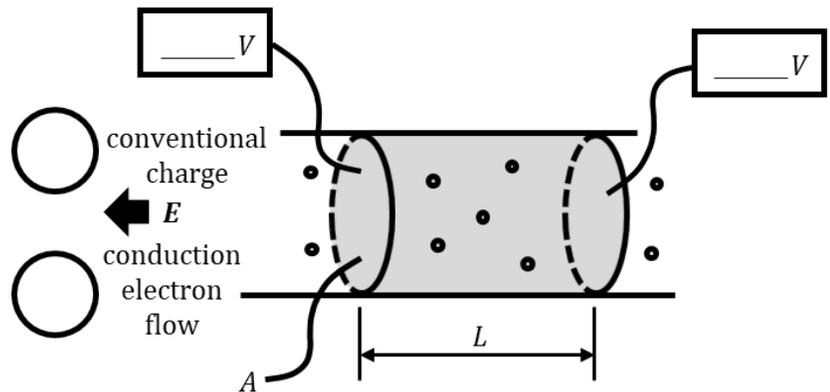
- A. Take your answer from Part J of Problem 1 and divide both sides of it by  $A$ . Write that result here.
  
- B. Recall that the current density  $J$  is defined as "current per area." Write an equation that expresses this definition.
  
- C. Use the transitive property to connect your Parts A and B answers to obtain a new expression for charge density.
  
- D. Show that your Part C answer is dimensionally consistent.

Okay, moving on... Resistivity is the property of a material that tells us how strongly it resists the flow of charge, while conductivity is the property of a material that tells us how easily charge flows through it.

- E. What are the symbol AND typical unit for resistivity?
  
- F. What are the symbol AND typical unit for conductivity?
  
- G. What is the mathematical relationship between resistivity and conductivity?
  
- H. In the unit you answered in Part E, the resistivity of zinc (Zn) has a magnitude of  $6.0 \times 10^{-8}$ . Why is this value NOT a surprise...for this material?
  
- I. Calculate the conductivity of zinc, based on the info in Part H.

A. Into the large circles in the figure, draw arrows to indicate the direction of conventional charge flow AND the direction of conduction electron flow.

B. Into the rectangles in the figure, indicate which portion of the cross-section has the higher potential  $V$  and which has the lower potential  $V$ .



C. Up to this point in the course, we have often used an equation that relates the separation between capacitor plates  $d$ , the electric field strength  $E$ , and the voltage  $V$  between plates. Write that equation, i.e.,  $V = ?$  here.

D. Now, the voltage difference  $V$  between the two cross-sections of wire in the figure is related to the  $E$  field in the wire and the distance between cross-sections by the same equation as your Part C answer. Modify that equation to account for the variables we have in this problem.

E. If you need to, look up the equation for Ohm's law. Write the "no denominators" form of that equation here.

F. Use the transitive property to connect your Parts D and E answers.

G. Resistivity  $\rho$ , besides being the reciprocal of conductivity  $\sigma$ , is also defined as being equal to the electric field strength divided by the current density. Write that equation.

H. Solve your Part G answer for the electric field.

I. Write the equation that IS the definition of current density.

J. Substitute your Part I answer into your Part H answer.

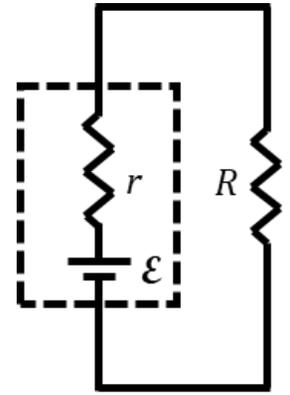
K. Substitute your Part J answer into your Part F answer.

L. Simplify your Part K answer to obtain an equation for the resistance of a wire that has particular physical properties. (Then, go outside and...PLAY. ☺)

UB, HW3, P4

Reference Video: "Maximizing Power for an External Resistor"  
YouTube, lasseviren1, DC CIRCUITS playlist

The figure shows a non-ideal battery of voltage  $\mathcal{E}$  and internal resistance  $r$  in a circuit having another resistor  $R$ . In terms of  $\mathcal{E}$ ,  $r$ , and  $R$ , find expressions for:



A. the total resistance  $R_{tot}$  of the circuit

B. the current  $I$  flowing through the circuit

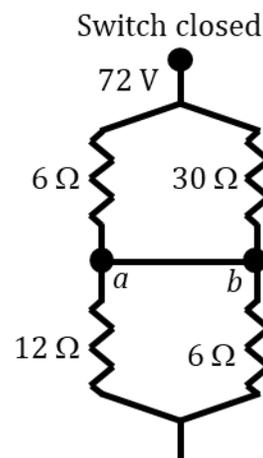
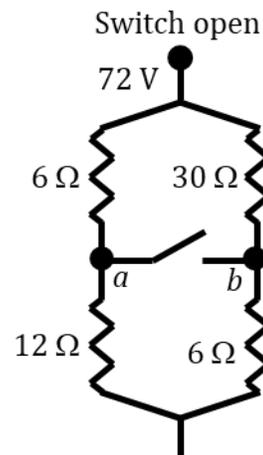
C. the power  $P$  consumed by the resistor  $R$

Use equations from Problems 1-3 to solve the following two problems. Show your work, include correct units, and use proper sig figs on your answers.

D. Household wiring is done with copper, which has a charge carrier density of  $8.491 \times 10^{28}$  carriers/m<sup>3</sup>. For a typical 20-A circuit, building codes require the use of 12-gauge wire (diameter = 2.05 mm). Given that each charge carrier has the elementary charge ( $1.60 \times 10^{-19}$  C/charge), determine the drift velocity of the conventional charges in such a circuit. Assume the current is DC (which it really isn't) and that the current is the maximum allowable value of 20.0 A.

E. Assume that the old-fashioned 60-W lightbulbs had tungsten filaments of length 1.98 m and thickness 0.0343 mm. If the conductivity of tungsten is  $8.93 \times 10^6$  ( $\Omega\text{-m}$ )<sup>-1</sup>, determine the resistance of the filament of such a lightbulb.

The figures are bridge circuits, which have a switch between Points *a* and *b*. The top figure has the switch open; the bottom figure has the switch closed. Neither figure shows an entire circuit; they show only the part of the circuit between a potential of 72 V (at the top of each figure) and ground (at the bottom).



A. In both figures, at the bottom, draw the symbol for ground.

B. By definition, what is the potential of ground?

C. When the circuit is open, it is a "left" and a "right" side in parallel, with each side having two resistors in series. Determine the following. Include units.

- i.  $R_{left} =$                       ii.  $R_{right} =$                       iii.  $R_{tot} =$                       iv.  $I_{tot} =$

D. Your Part C iv answer will divide left-and-right in inverse proportion to the left and right resistances. Determine these currents.

- i.  $I_{left} =$                       ii.  $I_{right} =$

E. Use your previous answers to determine the potentials at *a* and *b*.

- i.  $V_a =$                       ii.  $V_b =$

F. When the circuit is closed, it is essentially a "top" and a "bottom" in series, with each part having two resistors in parallel. Determine the following. Include units.

- i.  $R_{top} =$                       ii.  $R_{bot} =$                       iii.  $R_{tot} =$                       iv.  $I_{tot} =$

G. Determine the potential of the bridge region.                       $V_{bridge} =$

H. Use your previous answers to determine the currents (to two decimal places) in each resistor.

- i.  $I_{6left} =$                       ii.  $I_{12left} =$                       iii.  $I_{30right} =$                       iv.  $I_{6right} =$