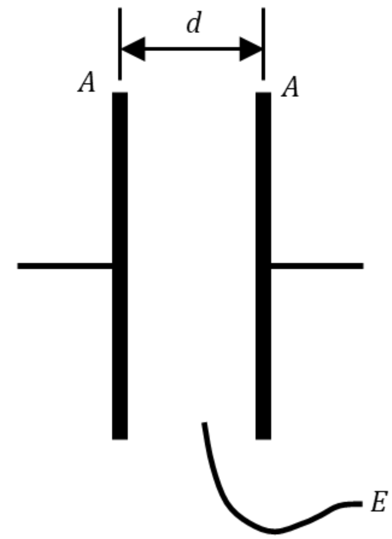
APPC, E & M: Unit B HW 1

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

UB, HW1, P1
Reference Video: "Basics of Capacitors (Part I)"
YouTube, lasseviren1, CAPACITANCE playlist

A. Capacitance is a measure of the charge-storage (and/or energy-storage) CAPACITY of two conductors that possess equal and opposite amounts of charge Q. The two conductors differ in electric potential by the amount V. Write the equation that IS the definition of capacitance.



B. Write the name AND symbol for the unit of capacitance.

The figure shows capacitor plates of area A, the region between plates of width d, and wires connecting the plates to the rest of the circuit. The capacitor is charged with a battery of voltage V, which moves charge from one plate to the other, resulting in the plates having equal and opposite charges, as well as an E field between plates.

C. In the figure, next to one plate, write the standard symbol (with sign!) that indicates a net charge on that plate. (Hint: It's a letter, usually an uppercase one.) Write a similar symbol on the other plate.

D. Next to your answers to Part C, write the standard symbols (with sign!) that indicate that charge is distributed over a given surface area. Also, write a series of signs along each plate to indicate that charge is distributed over each plate.

E. Write an equation that relates your symbols from Parts C and D together, in conjunction with one of the variables given in the figure. Don't write both (+) and (–) equations; one equation is plenty good.

F. Draw proper E field lines into the figure, in accordance with your work from Parts C and D.

G. Write the simple equation that relates the E field strength, the potential between plates (which grows until it equals the battery voltage V), and the plate separation d. (Hint: This equation is based on the fact that a potential difference V can be found by taking a path integral of the E field from one point to another, i.e., $|V| = \int \vec{E} \cdot d\vec{r}$.)

H. Combine your answers to Parts A, E, and G into a single equation for capacitance C, i.e., C = ? (Hint: The right side of this equation will have four variables: two in the numerator and two in the denominator.)

I. The strength of the E field at any point on the surface of a conductor conforms to a very simple equation based on the surface charge density at that point. This equation applies also to the strength of the uniform E field anywhere in the region between a capacitor's plates. Write that equation here. (Hint: You derived this equation in Problem 1 of UA, HW8.)

J. Combine your answers to Parts H and I to obtain an equation showing that capacitance C depends only on properties of the capacitor and one or more fundamental constants.

UB, HW1, P2
Reference Video: "Basics of Capacitors (Part II)"
YouTube, lasseviren1, CAPACITANCE playlist

Here, you will use Kirchhoff's so-called Loop Rule, which is a rule used for writing equations that relate to electric circuits. The basic Steps are:

I. We start at some point (any point) on the circuit's pathway.

II. We travel around the circuit, either clockwise (CW) or counter-clockwise (CCW), eventually returning to our starting point.

III. As we go around the circuit, we increase or decrease the voltage, as appropriate, whenever we encounter a circuit element.

IV. By the time we have returned to the starting point, the amount by which we have increased the voltage and the amount by which we have decreased the voltage will exactly cancel each other, leaving us with a net CHANGE in voltage of…zero.

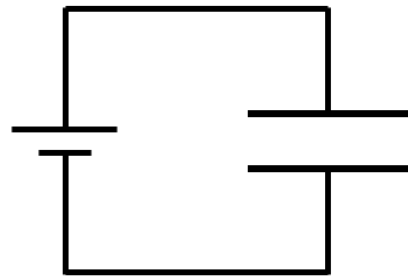


Fig. 1: Capacitor charging

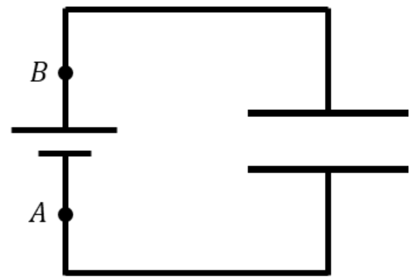


Fig. 2: Capacitor fully charged

Here are the Guidelines for the circuit elements of batteries and capacitors:

1. As we pass through a battery, if we are traveling in the direction of its (–) to its (+) terminal, the voltage increases (by $+V$). If we are going from its (+) to its (–) terminal, the voltage drops (by $-V$).

2. As we pass through a capacitor in the direction of its E field, the V drops (by $-\frac{Q}{C}$, i.e., by $-V$).
   When we pass through a capacitor going opposite the E field, the V increases (by $+\frac{Q}{C}$, i.e., by $+V$).

Your turn… A. In both Figures 1 and 2 above, label the (+) and (–) battery terminals, using one (+) and one (–) sign. Also, in both figures, label the battery with V and the capacitor with C.

B. Conventional (i.e., +) current will be flowing in Fig. 1. On the interior of the circuit, draw a nearly-continuous circular arrow-line (Yes, you will have to decide if it's CW or CCW…) to show the flow of conventional current. We say "nearly-continuous" because there is an insulator between the capacitor plates, and so NO current can flow through that…so your arrow should be discontinuous there.

C. In Figure 2 only: (1) Write a Q next to the charged capacitor, (2) draw a series of (+) and (–) signs on the plates of the now-charged capacitor, and (3) draw (AND label) the E field lines between the plates.

With reference to Fig. 2, we will now write out the four equations for this circuit that are possible using Kirchhoff's Loop Rule. The first one has been done for you, as an example. Note how this example conforms exactly to Steps I-IV and Guidelines 1 and 2, above. When you finish, you should see how all four equations are exactly equivalent to each other.

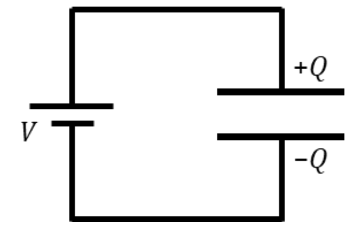
Example: Starting (and ending) at Pt. A in the CW direction: $+V - \frac{Q}{C} = 0$

D. Starting (and ending) at Pt. A in the CCW direction:

E. Starting (and ending) at Pt. B in the CW direction:

F. Starting (and ending) at Pt. B in the CCW direction:

UB, HW1, P3
Reference Video: "Energy Stored in a Capacitor"
YouTube, lasseviren1, CAPACITANCE playlist



In this problem, you will derive the equations for the potential energy U stored in the E field that builds up between the plates of a charged capacitor.

The general equation relating electric potential energy U, charge q, and electric potential V is $U = qV$. The reason we cannot use this equation as-is when we have a capacitor charged to a potential difference V due to a net charge on the plates Q (i.e., as in… $U = QV$) is because NOT all of the dq "bits" that (together) make up the total Q…require an equal amount of energy to move, when charging the capacitor. The first dq bits move rather easily, while the last few dq bits are much more difficult to move.

A. Rewrite the $U = qV$ equation so that it represents the tiny amount of energy dU required to move any given dq "bit."

B. Write the equation that IS the definition of capacitance.
(If you need a reminder, see Part A of Problem 1 of this HW set.)

C. Your Part B answer always applies, no matter what the charge Q on the plates and the voltage V between the plates might be, at any given time. Solve your Part B answer for V, i.e., V = ?

D. Substitute your Part C answer into your Part A answer.

E. To find the total energy U stored in the capacitor (due to electrical work W done by the battery on the circuit's charges), we need to "add up" all of the dU bits that you showed in Part D. Do that here.

F. Use your Part B answer (modified as needed), in conjuction with your Part E answer, to obtain TWO other simple equations that can be used to calculate the energy U stored in a capacitor.

UB, HW1, P4
Reference Video: "Changing the Capacitance of a Capacitor"
YouTube, lasseviren1, CAPACITANCE playlist

Here, you will walk through a line of reasoning about what happens when we change a capacitor's capacitance by modifying the separation between plates. The video goes through the case of charging a capacitor having original plate separation *d*, and then separating the plates to a distance of 2*d*. Here, we will reduce the distance, from *d* to ½ *d*. We will let the original quantities (when the capacitor is charged when the plate separation is *d*) be C_o, V_o, Q_o, E_o, and U_o. You will also need to reference the following equations as justification for your answers. (There might be more than one possible answer.)

I. $C = \frac{Q}{V}$

II. $C = \frac{\varepsilon_0 A}{d}$

III. $V = Ed$

IV. $U = \frac{Q^2}{2C}$

V. $U = \frac{1}{2}QV$

VI. $U = \frac{1}{2}CV^2$

Here we go…

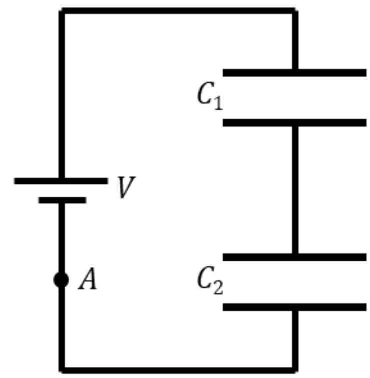
A. Let us charge the capacitor. Then, with the battery still connected into the circuit, we reduce the plate separation from *d* to ½ *d*.

How the new quantities compare to the original	Equation (AND/OR explanation) that justifies your answer
C_{new} = ____ C_o	
V_{new} = ____ V_o	
Q_{new} = ____ Q_o	
E_{new} = ____ E_o	
U_{new} = ____ U_o	

B. We now charge the capacitor again. Then, we disconnect the battery from the circuit, and then reduce the plate separation from *d* to ½ *d*.

How the new quantities compare to the original	Equation (AND/OR explanation) that justifies your answer
C_{new} = ____ C_o	
Q_{new} = ____ Q_o	
V_{new} = ____ V_o	
E_{new} = ____ E_o	
U_{new} = ____ U_o	

UB, HW1, P5
Reference Video: "Capacitors in Series"
YouTube, lasseviren1, CAPACITANCE playlist



In the figure, both capacitors are fully charged.

A. Place (+) and (−) signs at the appropriate points on the battery.

B. Place a series of (+) and (−) signs on the plates of capacitor C_1.

C. Refer to the figure. Notice the part of the circuit between the capacitors. (It is shaped like the capital letter "I" and, at this point, the top-horizontal portion of the I should have a series of signs along it. ☺) What is the net charge on this capital letter I?

D. Based on your Part C answer, place the correct number and type of sign on the bottom-horizontal portion of the I. Then, based on what you know about capacitors, put signs on the bottom plate of C_2.

E. Based on your work above, what MUST be true about the charge Q on each capacitor, if the capacitors are arranged in series?

F. Support your Part E answer by writing Q next to both C_1 and C_2.

G. Draw proper E field lines between the plates of C_1 and C_2.

H. Given that the battery voltage V is equal to the charge Q divided by the equivalent capacitance C_{eq}, write the Kirchhoff's Loop Rule equation that applies, starting at Point A and proceeding CW around the circuit and back to A. Start your equation by writing Q/C_{eq}, then refer to Problem 2 of this HW Set if you need a refresher.

I. Simplify your Part H answer to yield the equation for determining the equivalent capacitance C_{eq} for capacitors in series.