Electric Charge and Coulomb's Law

1.1 *Electricity and Magnetism Quantities*

UNIT of STUDY	QUANTITY	VARIABLE SYMBOL	UNIT AND SYMBOL
1	charge	q or Q	coulomb (C)
2	electric field	$\boldsymbol{E}$	newtons/coulomb (N/C) or volts/meter (V/m)
2	electric flux	Φ_E	volt-meter (V-m); also N-m^2/C
3	potential	V	volt (V)
3	potential energy	U	joule (J)
4	capacitance	C	farad (F)
5	conductivity	σ	siemen (S); also mho or 1/ohm-meter
5	current	I	ampere (A)
5	current density	$\boldsymbol{J}$	amps/square meter (A/m^2)
5	power	P	watt (W)
5	resistance	R	ohm (Ω)
5	resistivity	ρ	ohm-meter (Ω-m)
7	magnetic field	$\boldsymbol{B}$	tesla (T)
7	magnetic flux	Φ_B	weber (Wb)
8	inductance	L	henry (H)

1.2 *Electric Charge*

Electric charge is quantized. The smallest unit of electric charge that is able to be isolated is that on one electron or one proton; it is called the **elementary charge** e, and it has the value 1.60 x 10^{-19} C. All observable amounts of charge are integral multiples of e. The variable for electric charge (q or Q, either one) is derived from the fact that charge is quantized. Opposite charges (+ and –) attract each other; like charges (+ and +, or – and –) repel each other.

1.3 The Law of Conservation of Charge

The **law of conservation of charge** says that the algebraic sum of the charges in a closed region never changes. In some cases, charges may be created and destroyed but, for every (+) created or destroyed, there is simultaneously a (–) that is created or destroyed.

Examples of charge being destroyed are:

1. electron capture – An electron and a proton annihilate each other, both disappearing, with a neutron spontaneously coming into being

2. particle annihilation – One example is when an electron and a positron collide, both disappearing and producing one or more gamma rays

An example of charge being created is particle production, such as when an electron and a positron suddenly and simultaneously appear out of the void of space.

1.4 Conductors and Insulators

Substances in which a significant number of electrons are able to move freely throughout the material are called **conductors**. In conductors, the electrons that are free to migrate through the crystal lattice are called the **conduction electrons**. Metals are the most commonly-known conductors. In **insulators**, all electrons are confined to remain in the general vicinity of their particular atoms. Examples of insulators are wood, plastic, glass, air, and rubber.

1.5 Electrostatic Equilibrium

Electrostatic equilibrium is defined as the state in which there is no longer any net motion of charges. Charge could have moved prior to that, but the moving of charges is now done. In NO case (EVER!) does electrostatic equilibrium mean that an object or group of objects has a net charge of zero.

1.6 Coulomb's Law

The force between two motionless electric charges q is given by **Coulomb's law**. This law states that the **electric force** $\boldsymbol{F}$ (or **electrostatic force**, or **Coulombic force**, all are synonyms) exerted by one charge q_1 on a second charge q_2 separated by a distance r is given by:

$$\vec{\boldsymbol{F}}_{\mathbf{12}} = k\frac{q_1\, q_2}{r^2}\,\hat{\boldsymbol{r}} = \frac{1}{4\pi\epsilon_o}\frac{q_1\, q_2}{r^2}\,\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector directed from q_1 toward q_2.

By Newton's third law, the electric force $\boldsymbol{F}$ defined by Coulomb's law constitutes an action/reaction pair by each charge on the other.

The two forms of Coulomb's law given above are equivalent, with the constants being **Coulomb's constant** k and the **permittivity of free space** ε_o:

$$k = 9\ x\ 10^9\ \frac{N \cdot m^2}{C^2} \quad and \quad \epsilon_o = 8.85\ x\ 10^{-12}\ \frac{C^2}{N \cdot m^2}, \qquad where \quad k = \frac{1}{4\pi\epsilon_o}$$

As of 2018, the AP Physics C Exam uses only ε_o, i.e., it does not use k. We will freely use both.

To be clear, the unit vector $\hat{\boldsymbol{r}}$ can be eliminated when using Coulomb's law; one doesn't use it in finding the magnitude of the electric force F between two charges q. The magnitude of that force is given by:

$$F = k\frac{q_1\, q_2}{r^2} = \frac{1}{4\pi\epsilon_o}\frac{q_1\, q_2}{r^2}$$

Note that the F in the above equation is not bold-faced, nor does it have a vector arrow over it. This is because it represents a magnitude only. $\hat{\boldsymbol{r}}$ simply tells us in what direction the electric force F points, which is always along the straight line between two charges. That force acts either to pull the charges q towards each other (attraction, when q_1 and q_2 are oppositely charged) or push them away from each other (repulsion, when q_1 and q_2 are similarly charged). Like the unit vectors we've met before – namely $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, and\ \hat{\boldsymbol{k}}$ – the purpose of $\hat{\boldsymbol{r}}$ is merely to point.

When using Coulomb's law, the convention is that F is (–) for an attractive force and (+) for a repulsive force. This convention follows naturally from the equation itself: If q_1 and q_2 are oppositely charged, their product is a (–) number; if similarly charged (either both + or both –) their product is a (+) number.

If more than two charges are present, the net force on any one of them – due to the others – can be found using Coulomb's law and the **principle of superposition**, where the various forces are added vectorially.

Because charges q can have (+) or (–) signs, it is very easy to make mistakes when using Coulomb's law and superposition. In general, the best tactic is to simply find the magnitudes of the forces, ignoring the signs, and then draw a sketch that shows each force acting on the charge of interest, and in the correct direction, based on attraction or repulsion with each of the others. Then, break those vectors into two mutually perpendicular directions (i.e., the components) before Pythagorizing the summed components to obtain your final answer.

Here is a very important point, which you must keep in mind during your studies of electricity and magnetism over the next few months. You will be introduced to several equations by which you can calculate forces: specifically, electric forces and magnetic forces. (Coulomb's law is the first of these equations.) The forces will have effects on particles: particles that have charges, masses, and initial velocities. So, once you have computed the electric and/or magnetic forces, be prepared to use the equations and ideas of Newtonian mechanics to determine other quantities that you know and love: accelerations, final velocities, radii of curvature, and stopping distances. You have been warned.

1.7 Related Points

Physicists have long marveled at the similarity in structure between Coulomb's law of electric force and Newton's law of gravitational force. Both determine a force based on the product of a constant which is multiplied by two other quantities (each having the same unit as the other), and then divided by the square of the distance between the entities in question:

$$F = k\frac{q_1\,q_2}{r^2} \qquad and \qquad F = G\frac{m_1\,m_2}{r^2}$$

One big difference, of course, is that the gravitational force is always attractive, while the Coulombic force could be attractive or repulsive. The other big difference is that electric forces are much, MUCH, MUCH! stronger than gravitational forces...on the order of $10^{\text{thirty-something}}$ times stronger; that's a multiplicative factor of some number with thirty-something zeroes after it. (That's a lot, by the way.)

The intensity of light (and radiation, and sound) follows a somewhat similar quantitative pattern, in the sense that each is proportional to the square of the distance away from the source of the light (or radiation, or sound). For intensities, this observation has been explained by imagining concentric spheres of "something" emanating outward from the source. Similar "somethings" are believed to emanate outward in concentric spheres for electric charges q and for masses m; when two or more of those "somethings" interact, having emanated from separate and distinct sources, the electric or gravitational force is produced, so the story goes. But I digress...

1.8 The Shell Theorems

Shell theorem 1 for electrostatics (what I call the **outside-the-shell theorem**) states that a charged particle in the vicinity outside of a uniformly-charged shell is attracted or repelled as if all of the shell's charge were at a point at the shell's center.

Shell theorem 2 for electrostatics (what I call the **inside-the-shell theorem**) states that a charged particle located anywhere within a uniformly-charged shell experiences a net force of zero, due to the charges on the shell.

With both shell theorems, we assume that the particle's charge is too small to significantly alter the charge distribution of the shell.

1.9 Why Point Charges Attract All Neutral Objects

The following discussion will explain briefly why point charges are always attracted to neutral objects. This fact applies to all neutral objects, both conductors and nonconductors (i.e., insulators).

Consider first a neutral conductor; say, a metal sphere. Bring a point charge q near to the sphere, without touching it. It doesn't matter if q is (+) or (–); the same logic applies. We'll make it (+) here, just to illustrate the main idea. Some conduction electrons in the metal sphere are attracted to the $+q$ and migrate around the sphere to be closer to the $+q$. This gives the side of the sphere near the $+q$ a slight (–) charge and the opposite side of the sphere a slight (+) charge, although the overall net charge on the sphere is still zero. Then, because the electric force is stronger with decreasing separation, the attraction between $+q$ and the (–) portion of the sphere is greater than the repulsion between $+q$ and the (+) portion of the sphere. Thus, the neutral conducting sphere is attracted to the point charge.

The same result (namely, attraction) is obtained when the neutral object is a nonconductor, but for different reasons. In an insulator (say, a piece of plastic), no charges are free to move throughout the material; they must stay within a very specific locale. However, the electron cloud of each atom in an insulator can shift slightly to one side of the atom or the other, giving one side a slight (+) charge and the other a slight (–) charge, although the atom itself remains neutral. This is what happens whenever we bring our $+q$ near to a neutral insulator; its atoms become **polarized**, with the (–) electron cloud no longer centered on each nucleus, but rather shifted slightly toward $+q$. And again, the attractive force between $+q$ and the (nearer) negative portion of each atom outweighs the repulsive force between $+q$ and the (farther away) positive portion of each atom...and the neutral object is again attracted to the charged one.