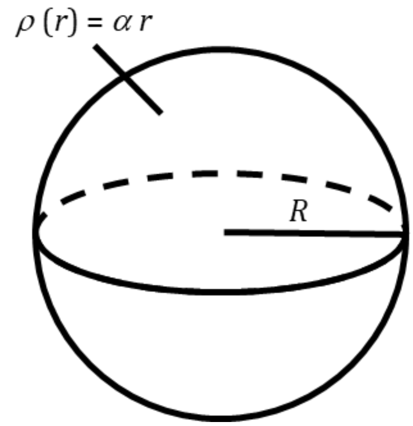
APPC, E & M: Unit A HW 6

Name: ______________________________

Hr: ___   Due at beg of hr on: __________

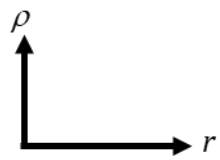
UA, HW6, P1

Reference Video: "Gauss's Law and Nonuniform Spherical Charge Distributions (Part I)"
YouTube, lasseviren1, GAUSS'S LAW playlist

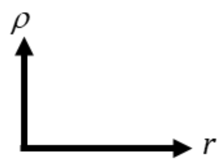


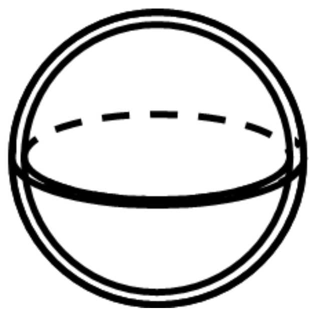
The figure shows a sphere of (+) charge having radius R. Previously (HW4, Prob. 3), we dealt with a sphere of uniform charge distribution ρ, i.e., ρ = constant. Here, the charge is NOT distributed uniformly; ρ changes linearly with distance from the center of the sphere, i.e., $\rho(r) = \alpha\, r$, where α is a positive constant.

A. Draw what a graph of ρ vs. r would look like, for $\rho(r)$ = constant.



B. Draw what a graph of ρ vs. r would look like, for $\rho(r) = \alpha\, r$.





Once again: Here, we are dealing with the case of your answer to Part B.

C. First, if $\rho(r) = \alpha\, r$, given what you know about ρ, what must be the units on α?

D. To analyze this charge distribution, we visualize the sphere as a very large number of extremely thin concentric shells. The figure to the right of the graphs above represents one of these extremely thin (and hollow) shells. Into that figure, draw it having radius r, thickness dr, and volume charge density ρ.

E. Suppose you could take this thin shell and cut it, as needed, to spread it out flat on a tabletop. It would no longer be a spherical shell, obviously, but there is a property of the original shell that is equal to the amount of the table's surface area that would now be hidden (or covered) by the spread-out "shell." In terms of the sphere's radius r, write the formula for the amount of table surface area that is now covered up, i.e., A = ?

F. Which variable in the shell-figure above would be equal to the height of the spread-out-on-the-table "shell"?

G. The volume of this very-thin THING that is spread over the table is extremely small, so let's call that volume dV. Use your Parts E and F answers to write an expression for dV.

H. But remember, this flat THING is actually a collection of charge, all of which was originally at some distance r away from the sphere's center. And, because the thickness/height (see your Part F answer) is very small, all of THAT charge has essentially the same ρ, even though ρ varies throughout the body of the sphere. With that in mind, use the given information and your Part G answer to write an expression for the small amount of charge dQ that makes up the volume dV. Hint: Your answer should be in terms of r and α.

UA, HW6, P2
Reference Video: "Gauss's Law and Nonuniform Spherical Charge Distributions (Part II)"
YouTube, lasseviren1, GAUSS'S LAW playlist

A. This is a continuation of HW6, Problem 1, so feel free to refer to that problem's figures and given info. In Part H of that problem, you wrote an expression for the small amount of charge dQ that makes up the volume dV in one particular thin shell of our entire sphere. Rewrite that expression for dQ here.

B. We now want to represent the total amount of charge Q that is enclosed within a solid sphere of radius $r < R$. With your Part A answer as a starting point, write the integral you will need to evaluate, i.e., Q = ?
Hints: Firstly, pull all constants outside the integral.
Secondly, you are NOT integrating over a closed surface, so your integral sign should NOT have a circle on it. Thirdly, the limits of integration are from $r = 0$ to $r = r$.

C. Evaluate the integral of Part B to obtain an expression for the total amount of charge Q that is enclosed within a sphere of radius $r \leq R$. (Again, recall that this is for a sphere having a charge distribution that varies linearly with distance from the sphere's center.)

D. Use your Part C answer and Gauss's law to find an expression for the E field for this particular charge distribution, for $r < R$.

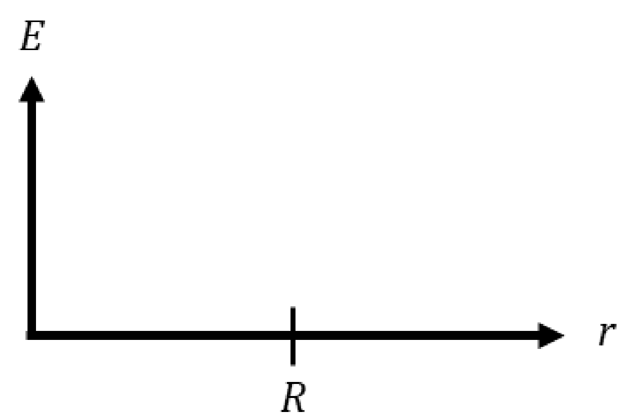
E. ε_o is the permittivity of free space. Given that Coulomb's constant k is $9\ x\ 10^9 \frac{N \cdot m^2}{C^2}$ and that $k = \frac{1}{4\pi\varepsilon_o}$, show that your Part D answer is dimensionally correct.

UA, HW6, P3
Reference Video: "Gauss's Law and Nonuniform Spherical Charge Distributions (Part II)"
YouTube, lasseviren1, GAUSS'S LAW playlist

A. Let's finish our work from Problems 1 and 2. Here, write an expression for the E field for $r > R$. Start again with your Part B answer from Problem 2, then evaluate that integral again (but slightly differently than you did in Part C) and, finally, use Gauss's law once more to find E = ? for $r > R$.

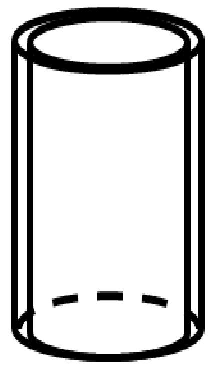
B. Show that your Part A answer above is dimensionally correct.

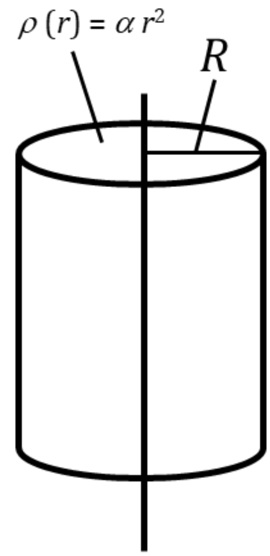
C. Graph your results from Problems 2 and 3. Above each of the two different parts of the graph, write a proportion or expression that indicates that you understand the behavior of E with respect to the distance r from the sphere's center.



UA, HW6, P4
Reference Video: "Gauss's Law and Nonuniform Cylindrical Charge Distributions (Part I)"
YouTube, lasseviren1, GAUSS'S LAW playlist

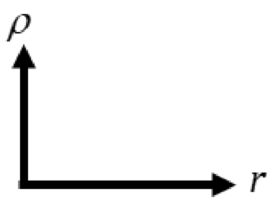
The figure at the far right shows a cylinder of radius R having a charge distribution that varies radially from the cylinder's axis, according to $\rho(r) = \alpha\, r^2$, where α is a positive constant.





A. If $\rho(r) = \alpha\, r^2$, given what you know about ρ, what must be the units on α?

B. Graph ρ vs. r.



C. To analyze this charge distribution, we visualize the cylinder as a very large number of extremely thin coaxial cylindrical shells. The figure above represents one of these extremely thin shells. Into that figure, draw it having radius r, thickness dr, height L, and volume charge density ρ.

D. Suppose you could take this thin shell and cut it, as needed, to spread it out flat on a tabletop. It would no longer be a cylindrical shell, but there is a property of the original shell that is equal to the amount of the table's surface area that would now be hidden (or covered) by the spread-out "shell." In terms of that shell's radius r and length L write the formula for the amount of table surface area that is now covered up, i.e., A = ?

E. Which variable in the shell-figure above would be equal to the height of the spread-out-on-the-table "shell"?

F. The volume of this very-thin THING that is spread over the table is extremely small, so let's call that volume dV. Use your Parts D and E answers to write an expression for dV.

G. But this flat THING is actually a collection of charge, all of which was originally at a distance r from the cylinder's center. And, because the thickness/height (see your Part E answer) is very small, all of that charge has the same ρ, even though ρ varies radially through the cylinder. With that in mind, use the given information and your Part F answer to write an expression for the small amount of charge dQ that makes up the volume dV. Your answer should be in terms of r, L, and α.

H. We now want to represent the total amount of charge Q that is enclosed within a cylinder of radius $r < R$. With your Part G answer as a starting point, <u>write the integral</u> you will need to evaluate, i.e., Q = ?
Hints: Firstly, pull all constants outside the integral. Secondly, you are NOT integrating over a closed surface, so your integral sign should NOT have a circle on it. Thirdly, the limits of integration are from $r = 0$ to $r = r$.

UA, HW6, P5
Reference Video: "Gauss's Law and Nonuniform Cylindrical Charge Distributions (Part II)"
YouTube, lasseviren1, GAUSS'S LAW playlist

This is a continuation of HW6, Problem 4, so feel free to refer to that problem's figures and given info.

A. Evaluate the integral of your Problem 4, Part H to obtain an expression for the total amount of charge Q that is enclosed within a solid cylinder of radius $r \leq R$. (Again, recall that this is for a cylinder of radius r having a charge distribution that varies according to r^2.)

B. Use your Part A answer and Gauss's law to find an expression for the E field for this particular charge distribution, for $r < R$.

C. Recalling that Coulomb's constant k is $9 \ x \ 10^9 \frac{N \cdot m^2}{C^2}$ and that $k = \frac{1}{4\pi\varepsilon_o}$, show that your Part B answer is dimensionally correct.

D. Now, we aim to write an expression for the E field for $r > R$. Start again with your Part H answer from Problem 4, then evaluate that integral again (but slightly differently than you did in Part A) and, finally, use Gauss's law once more to find E = ? for $r > R$.

E. Show that your Part D answer above is dimensionally correct.

F. Graph your results from Parts B and D. Above each of the two different parts of the graph, write a proportion or expression that indicates that you understand the behavior of E with respect to the distance r from the cylinder's center.