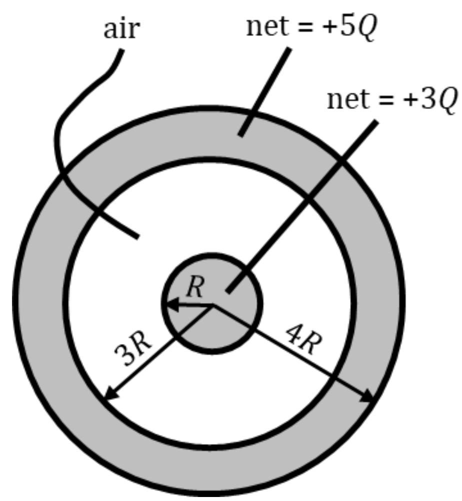
APPC, E & M: Unit A HW 5

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

UA, HW5, P1
Reference Video: "Gauss's Law and Concentric Spherical Shells (Part II)"
YouTube, lasseviren1, GAUSS'S LAW playlist

In Problem 5 of HW4, you tackled the situation shown in the figure:
A solid metal sphere of radius R is separated by an air space from a solid metal shell of inner radius $3R$ and outer radius $4R$. There is a total net charge of $+3Q$ on the innermost sphere and a total net charge of $+5Q$ on the outer shell. First, answer a few review questions from that problem...



A. What is the net charge (both magnitude and sign) that resides on the inner surface of the metal shell?

B. What is the net charge (both magnitude and sign) that resides on the outer surface of the metal shell?

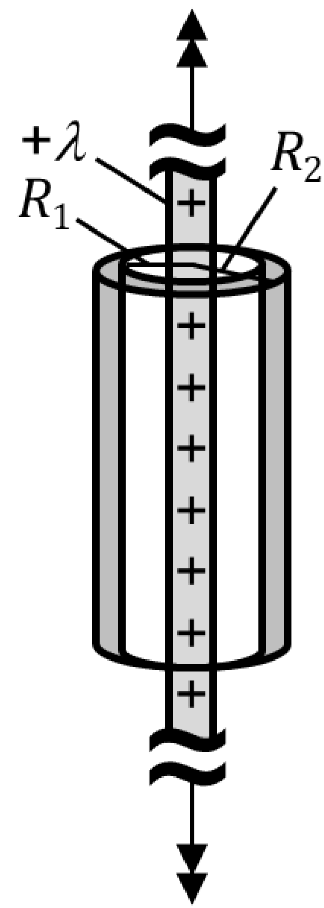
Now, let's deal with the σ on the various metal surfaces.

C. For σ ... ...what is its name? ...what are its units?

D. In terms of Q and R, derive expressions for: $\sigma_{\text{solid sphere surface}}$, $\sigma_{\text{shell inner surface}}$, and $\sigma_{\text{shell outer surface}}$.

UA, HW5, P2
Reference Video: "Gauss's Law and Coaxial Cables or Cylinders (Part I)"
YouTube, lasseviren1, GAUSS'S LAW playlist

The figure shows a coaxial cable that has a metal wire down the center having a charge per unit length $+\lambda$. Air surrounds the wire, and on the outside is a metal cylindrical shell of inner radius R_1 and outer radius R_2. The outer cylindrical shell has zero net charge.



A. Sketch a Gaussian cylinder with height h and radius $R_1 < r < R_2$, i.e., within the metal of the cylindrical shell. What is the E field at that radius?

B. Based on your answer to Part A, what is the total q_{enc} for that Gaussian surface?

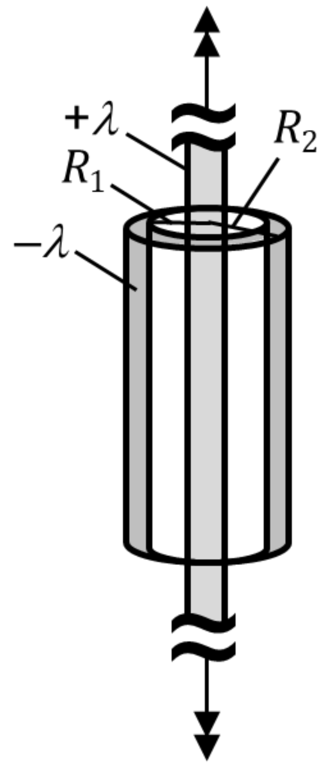
C. Based on the given info and your Part B answer, what is the charge per unit length along the inner surface of the actual metal shell, i.e., what is λ_{inner}?

D. Based on the given info and your Part C answer, what is the charge per unit length along the outer surface of the actual metal shell, i.e., what is λ_{outer}?

E. Convert your answers from Parts C and D into a σ_{inner} and a σ_{outer}.
Hint: It is easiest to use the h you defined in Part A; the h will then cancel and not appear in your final answers.

UA, HW5, P3

Reference Video: "Gauss's Law and Coaxial Cables or Cylinders (Part II)"
YouTube, lasseviren1, GAUSS'S LAW playlist



The figure shows a coaxial cable that has a metal wire down the center having a charge per unit length $+\lambda$. Air surrounds the wire, and on the outside is a metal cylindrical shell of inner radius R_1 and outer radius R_2. The outer cylindrical shell has a net charge per unit length of $-\lambda$. This means that the total amount of charge $+Q$ for some length L of the wire would be found by taking $+Q = +\lambda L$. Furthermore, the total amount of charge $-Q$ for some length L of the outer shell would be found by taking $-Q = -\lambda L$, and the net charge on the entirety of both parts of the coaxial cable is…zero. (This problem differs only slightly from what you just did in Problem 2.)

A. Sketch a Gaussian cylinder with height h and radius $R_1 < r < R_2$, i.e., within the metal of the cylindrical shell. What is the E field at that radius?

B. Based on your answer to Part A, what is the total q_{enc} for that Gaussian surface?

C. Based on the given info and your Part B answer, what is the charge per unit length along the inner surface of the metal shell, i.e., what is λ_{inner}?

D. Based on the given info and your Part C answer, what is the charge per unit length along the outer surface of the actual metal shell, i.e., what is λ_{outer}?

E. Convert your answers from Parts C and D into area charge densities, i.e., an σ_{inner} and a σ_{outer}.
Hint: It is easiest to use the h you defined in Part A; the h will then cancel and not appear in your final answers.

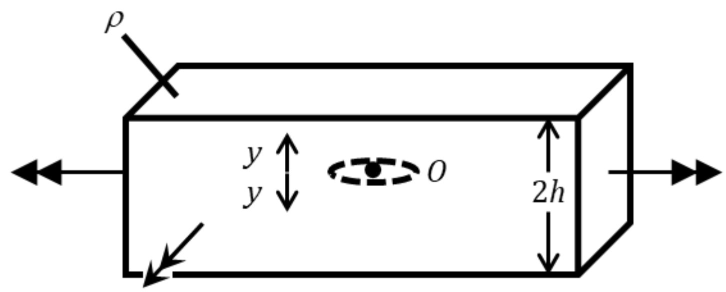
F. Imagine now another Gaussian cylinder having $r > R_2$. Look at the given figure again and decide what q_{enc} is, for that Gaussian cylinder. Write that answer here.

G. Based on your answer to Part F, Gauss's law requires that the magnitude of the electric field at any distance outside a coaxial cable (that is charged as shown in the figure) must be…

FYI: Your answer to Part G is one of THE important reasons that coaxial cables were invented. ☺

UA, HW5, P4
Reference Video: "Gauss's Law and the Electric Field Due to a Slab of Charge"
YouTube, lasseviren1, GAUSS'S LAW playlist

The figure shows an "infinite" slab having a uniformly-distributed (+) charge throughout its volume. The slab continues outward in every horizontal direction for a very great distance. (This could not be easily conveyed in the figure.) The volume charge density is given by ρ, and the slab has a finite thickness of $2h$. Point O lies within the "centermost slice" of the slab, i.e., Point O is a distance h from both the top and bottom surfaces. Your task is to determine an expression for the electric field, within the slab, at any distance y above or below the slab's centermost slice, in the following way…



A. Firstly, how do we know that this slab is NOT a conductor?
(Or, said another way…What would be true about the charge distribution if it WERE a conductor?)

B. From symmetry arguments, what must the magnitude of the E field be, at Point O?

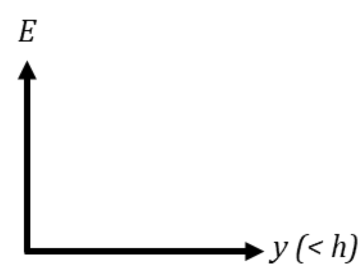
C. Into the figure, draw a vertical Gaussian cylinder, centered on Point O, and having a total height of $2y$. The dashed ellipse around Point O is intended to help you in this endeavor, by acting as the centermost slice of the Gaussian cylinder you draw. (You're welcome.) Both heights y are also shown in the figure; again, to help you. (Again: You're welcome. ☺) Label your cylinder's radius as r and the area of either its top or bottom as A.

D. Decide if there is electric flux Φ_E through these portions of your Gaussian cylinder: (circle)

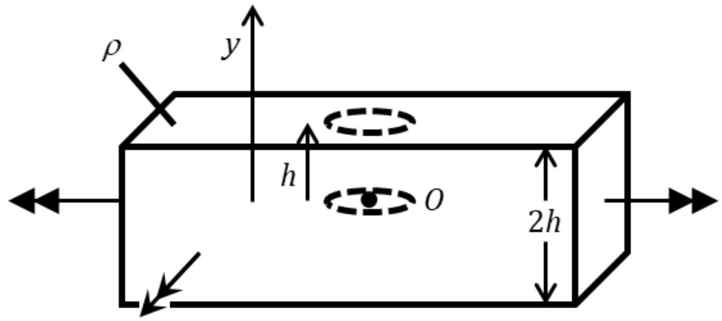
-- cylinder cap above Point O	YES	NO	-- cylinder wall above Point O	YES	NO
-- cylinder cap below Point O	YES	NO	-- cylinder wall below Point O	YES	NO

E. Now, use Gauss's law and your answer(s) to Part D to determine an expression for the electric field E within a slab having a volume charge density of ρ, at a distance y above (or below, same thing) the centerline of the slab (but within the slab, not outside it). Hint: Be VERY careful with the "area" term on the right side of the Gauss's law equation, or you'll screw up and get the wrong answer… ☹

F. Based on your Part E answer, make a simple graph showing how E varies with y. As you've done in the past, write a proportional relationship above the graph to show how E and y are related. (See HW4, Problem 2, Part G if you are confused about what to do here.)



UA, HW5, P5
Reference Video: "Using Gauss's Law to Find the Electric Field Due to a Plane of Charge"
YouTube, lasseviren1, GAUSS'S LAW playlist



The figure shows the same "infinite" slab of Problem 4. As before, the slab has a uniformly-distributed (+) charge throughout its volume; it continues outward in every horizontal direction for a very great distance; its volume charge density is given by ρ; and its thickness is $2h$. Also as before, Point O lies within the slab, equidistant from the top and bottom surfaces. This time, however, we want an expression for the E field OUTSIDE the slab, at any distance y above or below the slab's centermost slice, where y is always greater than h. Okay, here we go…

A. Into the figure, draw a vertical Gaussian cylinder, centered on Point O. The dashed ellipses around Point O and above it are intended to help you draw this cylinder, showing how it starts at the centerline by Point O (that's the first ellipse) and then how it extends upward, piercing the slab's surface (that's the other ellipse). Continue drawing the cylinder upward until it has the height y, shown in the figure. (So a third ellipse way up there would be great…) Label your cylinder's radius as r and the area of one of its caps as A. Unlike the cylinder in Problem 4, let's draw this cylinder only ABOVE Point O, not both above and below O, as you did in Problem 4. (But as long as you used Gauss's law correctly, having the cylinder both above and below would make no difference).

B. Once again: From symmetry arguments, what must the magnitude of the E field be, at Point O?

C. Decide if there is electric flux Φ_E through various portions of your Gaussian cylinder. Hint: Be very careful…You will need to consider your Part B answer as you decide which answers to circle.

-- cylinder cap at Point O	YES NO	-- cylinder wall within the slab	YES NO
-- cylinder cap at the top	YES NO	-- cylinder wall above the slab	YES NO

D. Now, use Gauss's law and your answer(s) to Part C to determine an expression for the electric field E outside a slab having a volume charge density of ρ, at a distance y above (or below, same thing) the centerline of the slab.

E. Here, combine your answer to Part F on Problem 4 with your answer to Part D above. Make a simple graph showing how E varies with y, both within the slab (i.e., for $y < h$) and outside the slab (i.e., for $y > h$). Essentially, first, you will re-copy your work from Part F on Problem 4 into the leftmost part of the graph here. Then, you will graph your result from Part C in Problem 5. Above each of the two different parts of the graph, write a proportion or expression that indicates that you understand the behavior of E with respect to the distance y from the slab's centerline.